‘OIL’IGOPOLY EXPLORATION: WHY SMALLER PRODUCERS EXPLORE MORE [#]

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ABSTRACT: The ‘oil’igopoly theory of oil production with fixed reserves predicts that firms with larger reserves will extract a larger quantity but a smaller proportion of their reserves. While this theory is supported when looking at production data, it is not supported when looking at changes in proven reserves data. This paper develops a theory of ‘oil’igopolistic oil exploration to explain trends observed in the world oil industry over the past fifty years. The ‘oil’igopoly theory of oil exploration predicts that firms with smaller proven reserves will do more exploration than firms with larger proven reserves, as well as reproducing the predictions of the ‘oil’igopoly oil production model. These predictions are consistent with international production and reserve data in the post-World War II era.

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1. INTRODUCTION

The theory of ‘oil’igopoly, developed by Loury (1986) and Polasky (1992), has the simple yet elegant prediction that firms holding larger reserves of oil tend to produce larger quantities of oil but a smaller proportion of their reserves in each period. Polasky found this prediction to be empirically supported when looking at a cross-section of oil producing nations. An important implication of this theory is that smaller firms will exhaust their reserves at a faster rate than larger firms. However, our own analysis indicates that a very curious thing has happened in the post World War II era. While larger oil producers have indeed been producing both larger quantities and a smaller proportion of their reserves in each year, it is smaller producers who have tended to see the largest growth rates in proven reserves. Since smaller producers use up their existing proven reserves at a faster rate than larger producers, the data suggest that smaller producers have made more discoveries of oil over these five decades than have larger producers. But why would smaller producers tend to do more exploration? While the Loury-Polasky theory of ‘oil’igopoly can explain the pattern of production relative to proven reserves, it cannot explain why smaller firms would do more exploration since there is no exploration in those models.

This paper develops an ‘oil’ogopolist theory of oil exploration to answer this question. To be successful, such a theory has to reproduce the Loury-Polasky result that larger firms produce a smaller proportion of their proven reserves and to explain why smaller firms do more exploration. We differentiate between proven reserves and unproven reserves. Proven reserves are those reserves for which exploration has already demonstrated the existence of an economically viable deposit. Unproven reserves are those reserves that the geologic indicators suggest exist, but which have not yet been discovered, or transformed into proven reserves, through exploration. Since exploration costs are on the order of millions of dollars for a well drilled on land to tens of millions of dollars for a well drilled at sea, exploration adds an important element to the theory of 'oil'ogopoly.[1]

The model we develop is a discrete time dynamic game in which heterogeneous firms in each period choose the amount of production and exploration. The crucial strategic function played by exploration in this model stems from the fact that to be produced, reserves must first be discovered. Since discovery costs are sunk once exploration occurs, firms gain a strategic advantage vis-à-vis their competitors by transferring reserves from an unproven to a proven status. Having lowered its marginal costs of future production, a firm has a credible threat to its rivals that it will produce a larger quantity in the next period. Hence, this threat reduces the future output of one's rivals. The strategic advantage conveyed by exploration is similar to that obtained from an increase in plant capacity, or R&D research to lower production costs in the industrial organization literature (e.g., Dixit 1980, 1986, Fudenberg and Tirole 1994, Bulow, Geanakopolis and Klemperer 1985). This strategic aspect of exploration leads us to model the game using subgame perfection as the equilibrium concept. This differs from the Loury-Polasky theory of 'oil'igopoly, which solved only for the Nash equilibrium. Eswaran and Lewis (1986)

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[1] The American Petroleum Institute reports that average drilling costs in the United States to be around seven hundred thousand dollars for an onshore well and over twelve million dollars for an offshore well in 2002.

showed that when firms possess well defined property rights, as in the Loury-Polasky model, that the two equilibrium concepts produced similar results.[2] However, as exploration belongs to the class of strategic investment models, subgame perfection is the appropriate equilibrium concept. Thus, the game is solved by backwards induction. Given that an exhaustible resource market exhausts the resource in the final period, this means that the game must be at least three periods long in order to see the strategic effects.

Like the Loury-Polasky theory of 'oil'igopoly, we find that firms holding larger proven reserves extract a larger quantity but a smaller proportion of their reserves in each period prior to exhaustion. We also find that this relationship holds for unproven reserves. The reason for these results is similar to the logic in the Loury-Polasky model. Larger firms produce a smaller proportion of their reserves because an increase in the output, which depresses the price, has a greater effect on their revenues than for a smaller producer. Here, this effect is amplified by the strategic advantages of holding larger reserves.

We find smaller firms doing more exploration than larger firms for two reasons. First, larger firms already have a credible commitment device to signal to rivals that they will produce a larger quantity in subsequent periods, so they do not need to use exploration for this purpose. For example, the largest oil producer, Saudi Arabia, held proven reserves that would last between seventy and eighty years at its current production levels during this period. Second, because firms holding larger reserves also tend to be larger producers they bear more of the cost of increases in output since the price reduction affects a larger quantity for these firms.[3] Thus larger firms not only do not need the additional reserves for a credible commitment, they also do not desire to hold larger levels of proven reserves. Thus, exploration is the primary instrument for

[2] Levhari and Mirman (1980) and Reignam and Stokey (1985) show that the open loop and subgame perfect equilibria differ significantly when stocks of the resource are common property.
[3] This is similar to the argument that a monopolist will do less innovating (Nordhaus 1969).

gaining a strategic advantage of small firms, while constraints on production are the primary instrument for gaining a strategic advantage of larger firms.

An important limitation of both the Loury-Polaksy model and the model we present is that firms face no uncertainty over their reserve holdings. Furthermore, we follow the Loury-Polasky assumption that firms do not face competition over their own reserves, whether proven or unproven. This assumption follows from evidence that most of the significant players in the world oil market are state-owned firms, which face little or no competition for access to the resource stocks within their own countries.[4] An important implication of these assumptions is that we can abstract from informational issues associated with exploration.[5]

This paper is closest in spirit to papers by Bulow and Geanakopolis (1983) and Hartwick and Sadorsky (1991). These papers were interested in the strategic effects from exploration from higher-cost stocks due to exploration's role as a commitment device. In both papers, firms produced in only two periods. In Hartwick and Sadorsky, firms in the first period choose both the level of exploration and production, but in the second period firms only produce from their remaining proven reserves – they do no further exploration. Thus, in their model, first period exploration affects one's rivals' subsequent profits, but does not affect one's rivals' subsequent behaviour, since in the second period all firms simply produce what remains of proven reserves. In Bulow and Geanakopolis, in each period firms produced from lower cost reserves and from a higher cost backstop technology.[6] Depletion of the lower cost reserves raised the future marginal costs of extraction from those reserves. However, the lower cost reserves were not exhausted in

[4] Fourteen of the top twenty (and nine of the top ten) oil firms by reserve holdings in 2003 are state owned firms ("A Survey of Oil," *The Economist*, April 30, 2005, p. 14).
[5] See Mason (1986), Isaac (1987), Polasky (1996), and Hendricks and Porter (1996) for models of information transmission in exploration. These models were developed with the U.S. system in mind, in which many firms compete for mineral rights. These models have focused on whether there is too little or too much exploration from an information gathering perspective and whether the timing of exploration has strategic information effects.
[6] A backstop technology is one that is available in large quantities at a constant marginal cost that exceeds the marginal cost of the exhaustible resource.

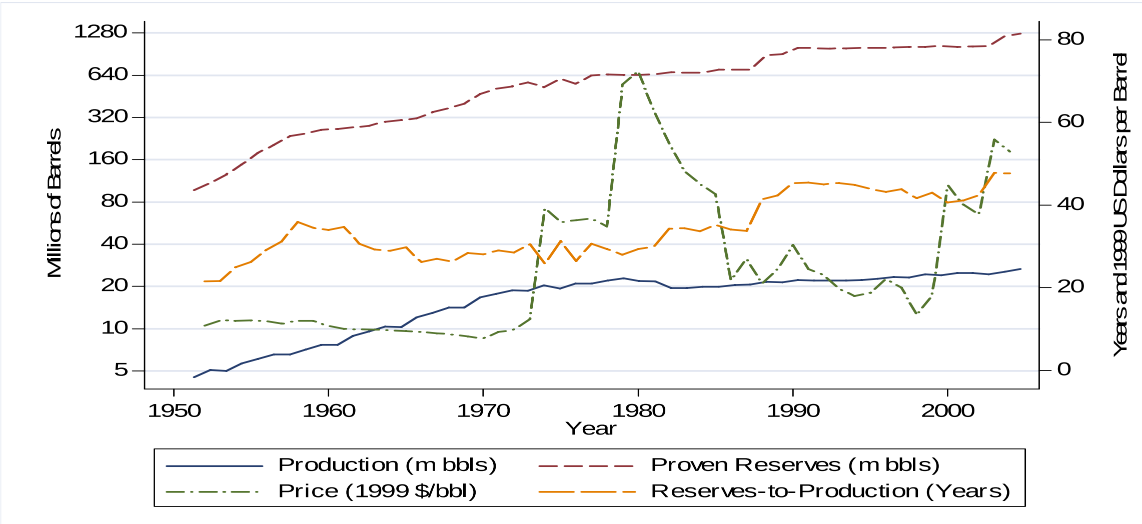
their model. Thus, current production affected subsequent behaviour as well as subsequent profits, but they did not consider the conditions under which firms would fully exhaust the resource. In our model, current exploration affects one's rivals' subsequent profits and their subsequent behaviour, since firms still get to choose how much to extract and explore and how much to leave for future extraction and exploration in each subsequent period. We also derive the conditions that must hold in order for a firm to rationally exhaust one or both types of its reserves. Like Hartwick and Sadorsky and Bulow and Geanakopolis, we find that firms behave strategically by over- exploring relative to an open loop benchmark. However, we also find that firms postpone some exploration to the final period in which they are active, since the proven reserves have lower marginal costs of extraction (e.g., Hartwick 1977).

The remainder of the paper is organized as follows. In Section 2, we briefly present some stylized facts about the world oil market over the period 1952-2002. Section 3 presents the basic equilibrium results of the model, beginning with what happens when there are two periods left before oil is exhausted. Section 4 derives the main results regarding strategic exploration by moving back one more period and asking how firms behave at that point, given the effects on rivals' subsequent behavior. It is in this section that we show why smaller firms are the ones most likely to be doing the most exploration. Section 5 concludes.

2. THE POST-WORLD WAR II OIL INDUSTRY

Figure 1 shows the world annual crude oil production and estimated proven reserves, measured on the left-hand-scale, and the annual real price and the reserves-to-production ratio, measured on the right-hand-scale, for the period 1952-2002. Both production and reserves were increasing rapidly during the period from the 1950s to the 1970s, doubling every dozen years or so. The two price shocks in the 1970s caused the increase in production to slow considerably relative to the pervious couple of decades. Reserves, however, have continued to increase, although the doubling time has dropped to almost twenty-five years.

FIGURE 1: WORLD PRODUCTION, RESERVES, AND PRICES 1952-2002



Notes:—The reserves data is from the *Oil & Gas Journal*. The Production data is from the *Energy Information Agency* (1960-2002) and from *World Oil* for other years. The price data is from *Energy Information Agency*.

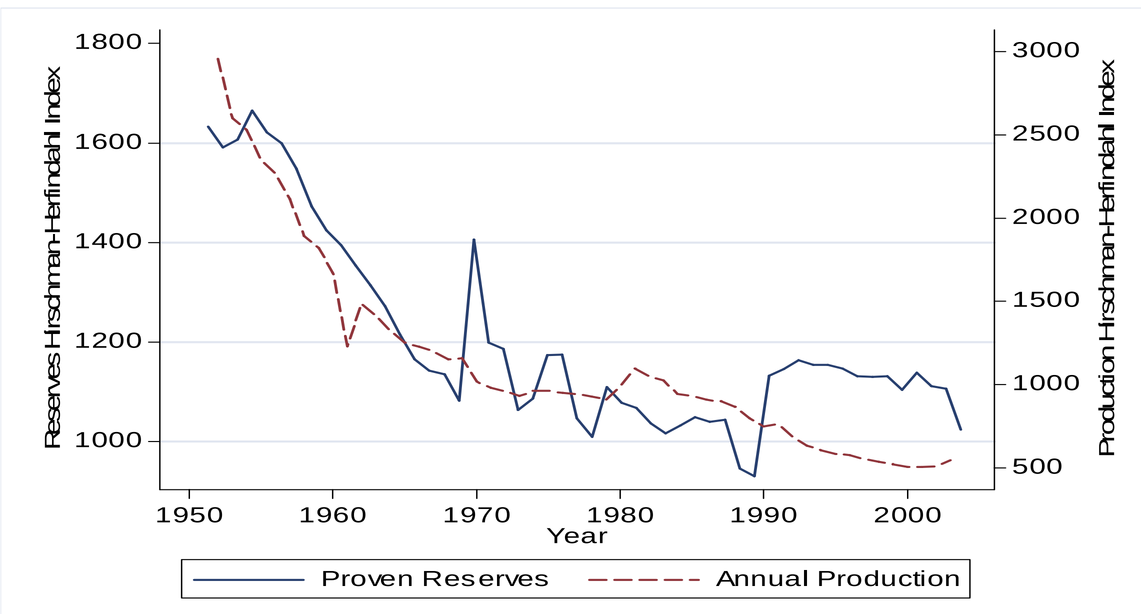
The reserves-to-production ratio, which measures the number of years current reserves would last at current production levels if there were no new exploration, rose rapidly in the 1950s, then with slack prices in the 1960s, hovered around thirty years worth of reserves, only to rise again to around forty years after the two price shocks of the 1970s. Note that while production increased five-fold over this period, proven reserves increased ten-fold.[7]

Figure 2 shows the Hirschman-Herfindahl concentration index for proven reserves and for annual production over the same five decades. By 2002, the concentration index for production dropped to less than twenty percent of its 1952 level. During this same period, the concentration index for reserves dropped to approximately sixty percent of its 1952 level. While this trend is

[7] We do not plot these, but both world per capita income and the population continued to rise post-1980 at similar rates as in the pre-1980s period. Thus the amount of oil consumed per capita declined after 1980 and the amount of oil consumed per dollar of GDP declined after 1970.

most pronounced in the 1950s and 1960s, for production the trend has continued to decline even in the 1980s and 1990s. The concentration index for reserves is also much more sporadic, as one might expect, with a large spike in the early 1970s with the discoveries in the North Sea and Alaska, and another large spike in the late 1980s with discoveries in the Middle East. Thus both reserves and production are becoming less concentrated. While the trends in figure 1 could have been occurred in competitive markets, the trends in figure 2 suggest that an oligopoly explanation is in order.

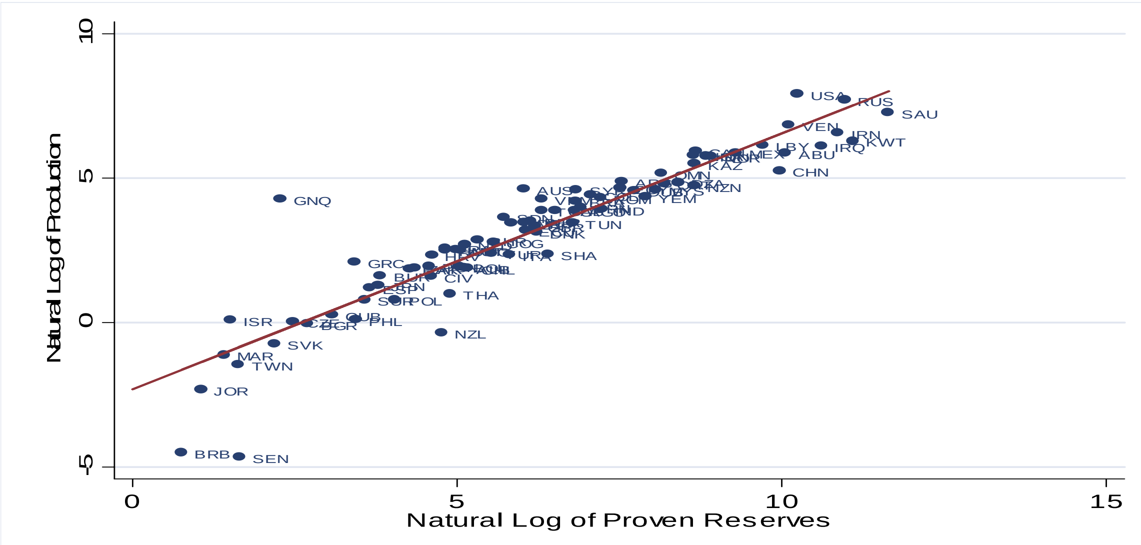
FIGURE 2: HIRSCHMAN-HERFINDAHL INDEXES OF CONCENTRATION IN CRUDE OIL PRODUCTION AND RESERVES, 1952-2002



Notes:—The reserves data is from the *Oil & Gas Journal*. The Production data is from the *Energy Information Agency* and from *World Oil*. The Hirschman-Herfindahl Index is calculated as $H_t = \Sigma_{i=1}^{n}(100s_{i\tau})^2$, where $s_{i\tau} = q_{it}/Q_t$ is the share of world production by country i in period t for production, and $s_{it} = R_{it}/R_t$ is the share of world oil proven reserves held by country i in period t.

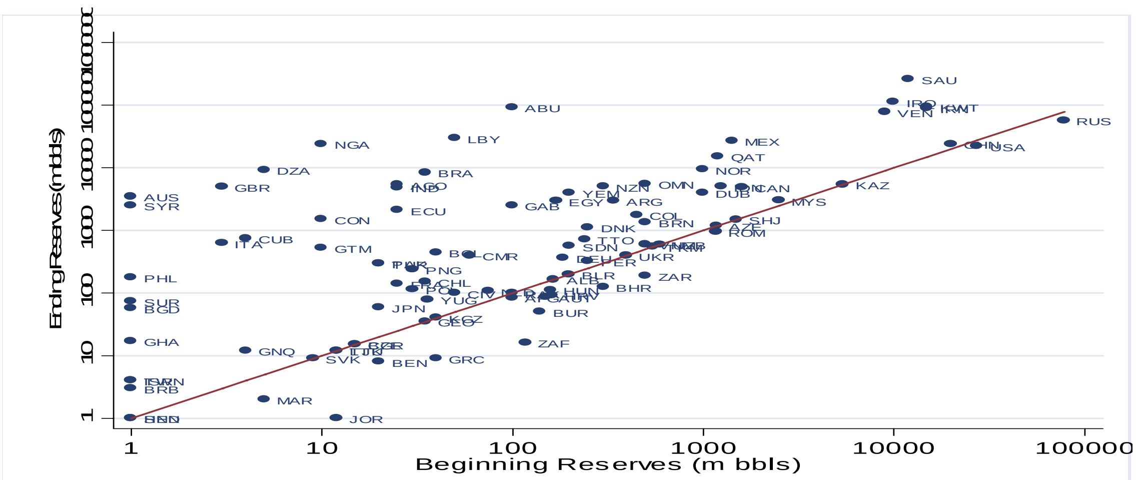
Figure 3 shows the relationship between production and reserves using average data by country for the period 1952-2002. While the analysis in figure 3 is a much simpler version of the analysis conducted by Polasky (1992), it supports his conclusion that larger firms produce larger quantities, but a smaller proportion of their reserves, as predicted by the theory of 'oil'igopoly.

FIGURE 3: PRODUCTION AND PROVEN RESERVES, COUNTRY AVERAGES 1952-2002



Notes:—Reserves data is from the *Oil & Gas Journal*. Production data is from the *Energy Information Agency* and from *World Oil*. The ordinary least squares regression line is $ln(q) = -2.30\ (0.27) + 0.88\ (0.04)\ ln(R)$ (standard errors in parentheses). The regression uses country averages for 82 countries over the period 1952-2002. The adjusted R^2 is 0.85.

FIGURE 4: ENDING PROVEN RESERVES AND INITIAL PROVEN RESERVES, 1952-2002



Notes:—Reserves data is from the *Oil & Gas Journal*. The number of years between initial reserves and ending reserves differs across countries. Countries above the 45° line have ending reserves greater than or equal to beginning reserves.

Figure 4 shows that countries with smaller initial reserves tended to have higher rates of growth in their proven reserves over the period 1952-2002. While there is greater variation in the countries with smaller reserves, the percentage changes in reserves is highest for countries that initially had smaller reserves. To our knowledge, this fact has not been observed in the literature prior to this paper. The empirical observation in figure 4 is the main stylized fact that this paper seeks to explain. Figure 4, coupled with the results shown in figure 3 that the industry is becoming less concentrated and the result in figure 1 that the reserves-to-production ratio has been rising are all puzzles that cannot be explained by the existing theory of 'oil'igopoly, since it ignores exploration. We now turn to the model.

3. 'Oil'igopoly Exploration and Production

At the beginning of each period t, let n_t firms hold oil reserves. Proven reserves held by the i^{th} firm at the beginning of period t are denoted as R_{it}; unproven reserves are denoted as S_{it}. The stocks of proven and unproven reserves for all firms are common knowledge. In equilibrium, a subset $m_t \leq n_t$ of the firms will have exhausted their lower cost proven reserves, and only hold unproven reserves. In each period, r_t firms exhaust their proven reserves, and s_t firms exhaust their total reserves. Thus, the number of firms holding each type of reserves evolves according to

(1) $$n_{t+1} = n_t - s_t \quad \text{and} \quad m_{t+1} = m_t - r_t, \quad t = 1, 2, \ldots$$

We shall assume that all firms exhaust their reserves by period three (i.e., $m_4 = n_4 = 0$), since this is a sufficient number of periods in which to observe the effects that we describe. Thus, $\Sigma_{t=1}^{3} r_t = m_1$ and $\Sigma_{t=1}^{3} s_t = n_1$. For a given allocation of reserves of each type, the number of firms exhausting each type is endogenous. However, rather than deriving the equilibrium number of firms that exhaust each type of reserves in each period, we shall fix the numbers $\{r_t, s_t\}_{t=1,2,3}$, and derive the conditions on the reserve holdings that have to be satisfied in equilibrium in order for this to occur.

Each firm chooses a level of output q_{it} and a level of reserve additions w_{it} in each period, t = 1,2,3. The model is deterministic, so each unit of exploration yields a fixed quantity of reserve additions. Given the production and reserve additions choices made by firms in period t, the stocks of proven and unproven reserves held by the i^{th} firm evolve according to

$$R_{it+1} = R_{it} + w_{it} - q_{it}, \qquad i = 1,\ldots,m_t,\ t = 1,2,3, \tag{2}$$
$$S_{it+1} = S_{it} - w_{it}, \qquad i = 1,\ldots,n_t,\ t = 1,2,3.$$

Initial reserves held by firm i are denoted as R_{i1} and S_{i1}, respectively. Demand is assumed to be time invariant, although this is not crucial to the analysis. The price at time t is given by $P_t = P(Q_t)$, where $Q_t = \Sigma_{i=1}^{n_t} q_{it}$. The demand function $P(Q)$ has a finite choke price, $0 < P(0) < \infty$, and is downward sloping, $P'(Q) < 0$. Extraction costs at time t are $c_{it}(q_{it})$, where $c'_{it} > 0$ and $c''_{it} \geq 0$. Thus the only grade differential in the stocks is the difference between proven and unproven reserves.[8] Discovery costs are given by $d_{it}(w_{it})$, where $d_{it}(0) = d'_{it}(0) = 0$, $d'_{it}(w) > 0$ and $d''_{it}(w) > 0$ for $w_{it} > 0$.[9] We also assume that $P(0) > c'_{it}(0) + d'_{it}(0)$, so that physical exhaustion of both types of stocks is ensured. Below, we shall also assume that the demand and cost functions satisfy more stringent conditions to obtain existence, uniqueness, and comparative statics results. Profits and costs one period in the future are discounted at a common rate $\beta \in (0,1)$. Both extraction and discovery costs are indexed by period to allow for an exogenous rate of technological change.[10] Thus, $c'_{it}(x) \geq c'_{i\tau}(x)$ and $d'_{it}(x) \geq d_{i\tau}(x)$, for $\tau > t$ and $x \geq 0$.

Since the subgame perfect equilibrium requires the derivation of a value function for future returns for each firm, we begin by considering the problem faced by firm i in period two, given that it holds reserves R_{i2} and S_{i2}, where $R_{i2} + S_{i2} > 0$, and given that at least some firms rationally

[8] See Swierzbinskin and Mendelsohn (1989) for a model of grade differentials under competitive extraction and exploration.
[9] Note that these properties of the discovery cost function are satisfied when $d_i(w_{it}) = Aw_{it}^{\zeta}$, for $A > 0$ and $\zeta > 1$.
[10] While we focus on the exploration and production patterns in figures 1-4, the increasing production in the face of decreasing prices in the pre-1970 and mid-1980s to late 1990s suggests that technological change played an important role in this market. See Smulders and de Nooji (2003) for a model of endogenous technological change in exhaustible resource markets.

exhaust in period three, and no firms exhaust beyond period three. Below, we shall derive the conditions on the reserves that have to hold in order for all firms to exhaust by period three. Let $\boldsymbol{R}_2$ and $\boldsymbol{S}_2$ denote the vectors of stocks $\{R_{12},R_{22},\ldots,R_{m_2 2}\}$ and $\{S_{12},S_{22},\ldots,S_{n_2 2}\}$, respectively, at the beginning of period two by all firms, and let R_{-it} and S_{-it} denote the sum of reserves held by firms other than firm i at the beginning of period t. Thus, firm i's problem in period two (P2), taking the choices of all other firms as fixed, is to choose extraction and production $\{q_{i2}, q_{i3}, w_{i2}, w_{i3}\}$ to maximize

P2 $$V_{i2} = P(Q_2)q_{i2} - c_{i2}(q_{i2}) - d_{i2}(w_{i2}) + \beta[P(Q_3)q_{i3} - c_{i3}(q_{i3}) - d_{i3}(w_{i3})],$$

subject to the following constraints:

(3) $$q_{i3} \geq 0, \qquad i = 1,\ldots,n_2,$$
(4) $$R_{i2} + w_{i2} + w_{i3} - q_{i2} - q_{i3} \geq 0, \qquad i = 1,\ldots,n_2,$$
(5) $$S_{i2} - w_{i2} - w_{i3} \geq 0, \qquad i = 1,\ldots,n_2,$$
(6) $$w_{i2}, w_{i3} \geq 0, \qquad i = 1,\ldots,n_2,$$
(7) $$R_{i2} + w_{i2} - q_{i2} \geq 0, \qquad i = 1,\ldots,n_2.$$

Constraints (3) and (4) are feasibility constraints on production due to the exhaustible nature of the resource. Constraints (5) and (6) are feasibility constraints on exploration. Constraint (7) ensures that extraction in period two is feasible. Let α_i, λ_i, μ_i, θ_{it}, and ϕ_i denote the Kuhn-Tucker multipliers for the constraints (3)-(7), respectively.

Since the model is only interesting if some firms produce in each period, as there can be no strategic effects if this is not true, we assume that $n_1 > s_1 + s_2$, so that some firms produce for three periods. The first-order necessary conditions for maximization of P2 include (3)-(7) and

(8) $$\frac{\partial V_{i2}}{\partial q_{i2}} = P(Q_2) + P'(Q_2)q_{i2} - c'_{i2}(q_{i2}) - \lambda_i - \phi_i = 0, \qquad i = 1,\ldots,n_2,$$

$$(9)\qquad \frac{\partial V_{i2}}{\partial q_{i3}} = \beta[P(Q_3) + P'(Q_3)q_{i3} - c'_{i3}(q_{i3})] - \lambda_i + \alpha_i = 0, \qquad i = 1,\ldots,n_2,$$

$$(10)\qquad \frac{\partial V_{i2}}{\partial w_{i2}} = -d'_{i2}(w_{i2}) + \lambda_i - \mu_i + \phi_i + \theta_{i2} = 0, \qquad i = 1,\ldots,n_2,$$

$$(11)\qquad \frac{\partial V_{i2}}{\partial w_{i3}} = -\beta d'_{i3}(w_{i3}) + \lambda_i - \mu_i + \theta_{i3} = 0, \qquad i = 1,\ldots,n_2.$$

The marginal value of the proven reserves R_{i2} to firm i at the beginning of period two is $\lambda_i + \phi_i$ and the marginal value of unproven reserves to firm i at the beginning of period two is μ_i. The conditions (8) and (9) have the usual interpretation that the marginal profit from extraction in each period is equal to the marginal value of the remaining resource stock. Equations (10) and (11) reveal that a similar dynamic is at work with unproven reserves.

It follows immediately that $\lambda_i > 0$ and $\mu_i > 0$. Suppose not. Suppose that either $R_{it} > 0$, $S_{it} > 0$, or both are positive in period 3 when production has shut down. Since $P(0) > c'_{it}(0) + d'_{it}(0)$, it is profitable for a firm holding positive stocks to extract the last unit of either type of stock, which is a contradiction. Thus $\lambda_i > 0$ and $\mu_i > 0$, which implies that the constraints (4) and (5) hold with equality. Therefore, $q_{i3} = R_{i2} + S_{i2} - q_{i2}$ for any firm that continues to produce in period three, and $q_{i2} = R_{i2} + S_{i2}$ for those firms that exhaust in period two. However, the constraints (3), (6) and (7) may or may not be binding along the equilibrium path. As will become clear, constraint (7) is crucial to the story we tell about strategic behaviour.

Using (9) to eliminate the shadow value of proved reserves, λ_i, from (8) yields

$$(12)\qquad \pi_1^{i2}(q_{i2}^*,Q_{-i2}) - \beta\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*,Q_{-i3}) = \phi_i + \alpha_i \qquad i = 1,\ldots,n_2,$$

where $\pi_1^{it}(q_{it}^*,Q_{-it}) \equiv P(Q_t) + P'(Q_t)q_{it}^* - c'_{it}(q_{it}^*)$ is the equilibrium marginal profit from extraction in period t, given that discovery costs are sunk and holding the output of all other firms, Q_{-it}, constant. When the constraints (3) and (7) do not bind, so that the multipliers ϕ_i and α_i are each zero, (12) shows that that marginal extraction profits are equal in present value, which is Hotelling's (1931) rule for an oligopolist (e.g., Salant 1976, Loury 1986, or Polasky 1992). When $\phi_i > 0$ but the constraint (3) is not binding, so that the extraction constraint (7) is binding but $\alpha_i = 0$, this forces the Hotelling condition to reflect the increase in extraction costs due to having to extract only from unproven reserves in period three, so that $\pi_1^{i2}(q_{i2}^*, Q_{-i2}) > \beta\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3})$. This means that the value of the marginal reserves declines in present value when (7) is binding. In contrast, when the firm chooses not to produce in period three ($\alpha_i > 0$), the condition (12) implies that $\pi_1^{i2}(R_{i2} + S_{i2}, Q_{-i2}) > \beta\pi_1^{i3}(0, Q_{-i3})$, so the firm does better by exhausting in period two than by taking some reserves into period three.

Similarly, (10) and (11) provide an intertemporal optimization condition for reserve additions:

$$(13) \qquad d'_{i2}(w_{i2}^*) - \beta d'_{i3}(S_{i2} - w_{i2}^*) = \phi_i - \theta_{i2} - \theta_{i3}, \qquad i = 1,\ldots,n_2.$$

We have substituted in from constraint (5) to write this entirely in terms of period two exploration. When the period two production constraint (7) is not binding and exploration is positive in each period, (13) says that the present value of marginal exploration costs is equal across time. When $\phi_i > 0$, so that the period two production constraint is binding, then the present value of marginal exploration costs is declining over time. This occurs for the same reason that marginal profits in (12) are rising at less than the rate of interest when (7) binds.

Our first result, which follows the assumption that $d'_{it}(0) = 0$, is that exploration is positive in both periods two and three, so long as production occurs in both periods:

Proposition 1: If firm *i* has positive quantities of unproven reserves at the beginning of period two and rationally exhausts in three periods, it will explore in both periods two and three.

Proof: This proposition is proven in the appendix.

Proposition 1 shows two things. First, when marginal exploration costs are increasing in the rate of exploration, that the firm will spread exploration across all periods. Second, because unproven reserves have higher marginal costs to extract than unproven reserves, each firm will produce from the high cost reserves in the final period in which it operates, even given the strategic advantages of early transformation into proven reserves. This was assumed not to occur in Hartwick and Sadorsky (1993), but proposition 1 shows that if the firm is given the choice of exploring in the last period, it will do so.

Proposition 1 implies that (13) can be written as

(14) $$d'_{i2}(w^*_{i2}) - \beta d'_{i3}(S_{i2} - w^*_{i2}) = \phi_i, \qquad i = 1,\ldots,n_2.$$

Thus, marginal exploration costs are constant in present value when the constraint (7) is not binding, and fall in present value when the constraint (7) is binding. In what follows, we use the fact that when the constraint (7) is not binding, (14) implies that there exists a value of $w^*_{i2} = w_{i2}(S_{i2})$ such that

(15) $$d'_{i2}(w_{i2}(S_{i2})) \equiv \beta d'_{i3}(S_{i2} - w_{i2}(S_{i2})).$$

Note that $0 < w'_{i2}(S_{i2}) < 1$. Thus, as S_{i2} increases, $w_{i2}(S_{i2})$ increases, but at a rate less than one.

While proposition 1 eliminates all equilibria with zero exploration in either period two or three, there remain three possible outcomes for a firm that produces in one or both the two remaining periods, depending upon whether or not the constraint (7) binds if production occurs in period three, and on whether or not the firm produces in period three:

Case A: firm *i* explores and extracts in periods 2 and 3 and the constraint (7) does not bind.
Case B: firm *i* explores and extracts in periods 2 and 3 but the constraint (7) binds.

Case C: firm i exhausts in period 2.

Define

(16) $$\psi_i(q_{i2}) \equiv \pi_1^{i2}(q_{i2}, Q_{-i2}) - d'_{i2}(q_{i2} - R_{i2}) - \beta[\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) - d'_{i3}(R_{i2} + S_{i2} - q_{i2})],$$

where $w_{i2} = q_{i2} - R_{i2}$, and $q_{i3} = w_{i3} = R_{i2} + S_{i2} - q_{i2}$ from (4) and (7). Thus, from (8)-(11), $\psi_i(q_{i2}^*) = 0$ when (7) is binding. Similarly, let:

(17) $$\eta_i(q_{i2}) \equiv \pi_1^{i2}(q_{i2}, Q_{-i2}) - d'_{i2}(w_{i2}(S_{i2})) - \beta[\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) - d'_{i3}(S_{i2} - w_{i2}(S_{i2}))],$$

where $w_{i2} = w_{i2}(S_{i2})$ is given by (15), and $q_{i3} = R_{i2} + S_{i2} - q_{i2}$ from (4). Thus, $\eta_i(q_{i2}^*) = 0$ when (7) is not binding. The difference between (16) and (17) is the value of w_{i2}. When $\phi_i = 0$, $w_{i2}^* = w_{i2}(S_{i2})$, but when the constraint (7) is binding, $\phi_i > 0$ implies that $w_{i2}^* > w_{i2}(S_{i2})$, since (14) implies that w_{i2}^* is increasing in ϕ_i. When $q_{i2}^* = R_{i2} + w_{i2}(S_{i2})$, $\psi_i(q_{i2}^*)$ is identical to $\eta_i(q_{i2}^*)$.

We proceed by first deriving and interpreting the conditions that hold for a particular firm that produces in periods 2 and 3 to have a unique best response to what the remaining industry is doing. Then we find a set of conditions on the demand and cost function that ensures that the best-reply mappings of all firms contracts to a unique equilibrium.

Proposition 2: Holding constant the actions of all other firms, if firm i produces in both periods 2 and 3, then firm i's unique choice of extraction and exploration exists satisfying $\psi_i(q_{i2}^*) = 0$ and (14) when (7) binds, if and only if,

(18) $$\psi_i(R_{i2} + S_{i2}) < 0 \qquad \text{and} \qquad \psi_i(R_{i2} + w_{i2}(S_{i2})) > 0;$$

and satisfying $\eta_i(q_{i2}^*) = 0$ and (15) when (7) does not bind, if and only if,

(19) $\eta_i(R_{i2} + w_{i2}(S_{i2})) < 0$ and $\eta_i(0) > 0$.

Proof: (*i*) Uniqueness: Since uniqueness is easiest to prove, we begin with it. When (7) binds, so that (16) and (14) define the equilibrium, we see from (16) that

$$\psi_i'(q_{i2}) = \pi_{11}^{i2}(q_{i2}, Q_{-i2}) - d_{i2}''(q_{i2} - R_{i2}) + \beta[\pi_{11}^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) - d_{i3}''(R_{i2} + S_{i2} - q_{i2})] < 0, \tag{20}$$

where $\pi_{11}^{it}(q_{it}, Q_{-it}) \equiv 2P_t' + q_{it}P_t'' - c_{it}'' < 0$ in order for second order conditions to hold. Similarly, we see that the left-hand-side of (14) is strictly increasing in q_{i2}:

$$d_{i2}''(q_{i2} - R_{i2}) + \beta d_{i3}''(R_{i2} + S_{i2} - q_{i2}) > 0. \tag{21}$$

Thus, if $\psi_i(q_{i2}^*) = 0$ for some feasible q_{i2}^* and (14) holds, then q_{i2}^* is unique.

The proof for (17) proceeds similarly. Differentiating (17) with respect to q_{i2} yields

$$\eta_i'(q_{i2}) = \pi_{11}^{i2}(q_{i2}, Q_{-i2}) + \beta\pi_{11}^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) < 0. \tag{22}$$

Thus, if $\eta_i(q_{i2}^*) = 0$ for some feasible q_{i2}^* and (15) holds, then q_{i2}^* is unique.

(*ii*) Existence (sufficiency): In the case where (7) is binding, (16) and (14) describe the equilibrium. Feasibility requires that

$$R_{i2} + S_{i2} > q_{i2}^* = R_{i2} + w_{i2}^* > R_{i2} + w_{i2}(S_{i2}). \tag{23}$$

Combining (23) with the monotonicity condition (20), we see that the two conditions in (18) are sufficient to prove the existence of an equilibrium when (7) binds.

When (7) does not bind, the corresponding feasibility condition is

$$0 < q_{i2}^* < R_{i2} + w_{i2}(S_{i2}). \tag{24}$$

Thus, by the monotonicity condition (22), we obtain (19) as the sufficient conditions to ensure a unique equilibrium.

(iii) Existence (necessity): To prove that the conditions in (18) are necessary to obtain an equilibrium when the constraint (7) binds, suppose one of the conditions is not binding. Suppose that $\psi_i(R_{i2} + S_{i2}) > \psi_i(R_{i2} + w_{i2}(S_{i2})) > 0$. Then by (20), no feasible value of q_{i2}^* exists that satisfies $\psi_i(q_{i2}^*) = 0$. Similarly, when (7) does not bind, if $\eta_i(0) > \eta_i(R_{i2} + w_{i2}(S_{i2})) > 0$, no feasible value of q_{i2}^* exists such that $\eta_i(q_{i2}^*) = 0$. This completes the proof.

The economic interpretation of the conditions (18) and (19) are straightforward. Let us consider (18) first. The condition $\psi_i(R_{i2} + S_{i2}) < 0$ can be written as

(25) $$\pi_1^{i2}(R_{i2} + S_{i2}, Q_{-i2}) - d'_{i2}(S_{i2}) < \beta[\pi_1^{i3}(0, Q_{-i3}) - d'_{i3}(0)].$$

This means that it is profitable for firm i to carry some of its production forward to period three. Since this condition forms the boundary between the cases where production ends in period two and continues to period three, we summarize it in this proposition:

Proposition 3: If, and only if, $\psi_i(R_{i2} + S_{i2}) < 0$, firm i will produce in period three, rather than ending production in period two.

Proof: (i) Sufficiency: If the inequality in (25) holds, then firm i will hold some of its reserves for production in period three. (ii) Necessity: If the inequality in (25) is reversed, then firm i prefers to exhaust in period two, rather than holding some reserves to period three. This completes the proof.

Note that the boundary created by $\psi_i(R_{i2} + S_{i2}) = 0$ intersects the boundary created by $\psi_i(R_{i2} + w_{i2}(S_{i2})) = 0$ at $S_{i2} = 0$. This occurs because when $S_{i2} = 0$, $w_{i2}(0) = 0$.

The condition $\psi_i(R_{i2} + w_{i2}(S_{i2})) > 0$ in (18) can be written as (using (15))

(26) $$\pi_1^{i2}(R_{i2} + w_{i2}(S_{i2}), Q_{-i2}) > \beta\pi_1^{i3}(S_{i2} - w_{i2}(S_{i2}), Q_{-i3}),$$

which means that marginal profits in period two exceed those in period three in present value when production is $q_{i2}^{*} = R_{i2} + w_{i2}(S_{i2})$. It is this condition that causes the firm to increase its production from unproven reserves. When the constraint (7) is not binding ($\phi_i = 0$), the relevant conditions are (15) and (19). The condition $\psi_i(R_{i2} + w_{i2}(S_{i2})) < 0$ is simply the reverse of the inequality in (26). Since the present value of marginal profits in period two is less than the present value of marginal profits in period three when all reserve additions are produced, the firm wishes to keep some of these reserve additions for use in period three.

Finally, the condition $\eta_i(0) > 0$ can be written as

$$\text{(27)} \qquad \pi_1^{i2}(0, Q_{-i2}) \; > \; \beta\pi_1^{i3}(R_{i2} + S_{i2}, Q_{-i3}).$$

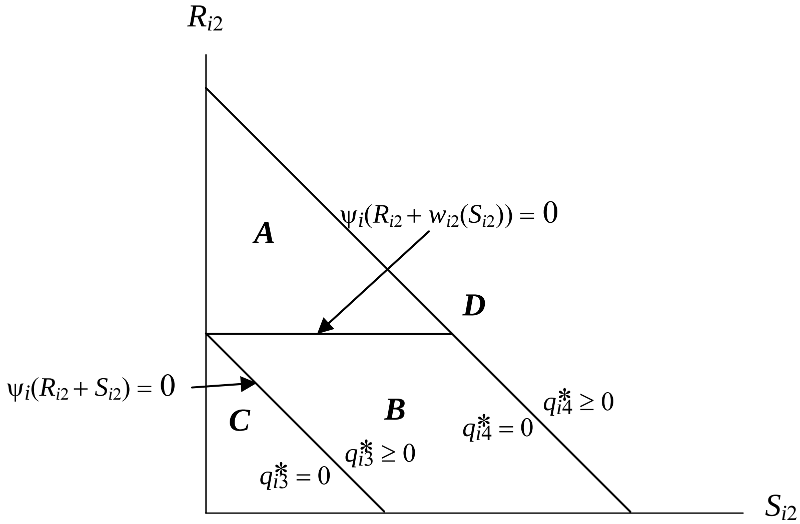
This condition says that the firm wishes to have some production in period two, rather than holding all production until period three.

It should be clear by now that the condition $\psi_i(R_{i2}+w_{i2}(S_{i2})) = \eta_i(R_{i2}+w_{i2}(S_{i2})) = 0$ serves to form a boundary separating the cases where (7) is and is not binding in R_{i2} and S_{i2} space. Totally differentiating this condition and solving for the slope of this locus yields

$$\left.\frac{\partial R_{i2}}{\partial S_{i2}}\right|_{\psi_i(R_{i2}+w_{i2}(S_{i2}))\,=\,0} = \frac{w_{i2}'\pi_{11}^{i2} - (1 - w_{i2}')\beta\pi_{11}^{i3}}{-\pi_{11}^{i2}},$$

where $\pi_{11}^{it} < 0$ by second order conditions. In general, this expression is ambiguous in sign. However, in the special case of linear demand, constant marginal extraction cost, and quadratic exploration costs, the slope of the $\psi_i(R_{i2}+w_{i2}(S_{i2})) = 0$ locus is zero. Since $\partial\psi_i(R_{i2}+w_{i2}(S_{i2}))/\partial R_{i2} = \pi_{11}^{i2} < 0$, an increase in R_{i2} causes the constraint (7) not to bind. This locus is shown in Figure 5 as the boundary where $q_{i2}^{*} \leq R_{i2} + w_{i2}(S_{i2})$.

Figure 5: Equilibrium Exploration and Exhaustion for Firm *i*



Notes—The areas *A* and *B* correspond to the areas where the constraint (7) is not binding and is binding, respectively, and firm *i* produces in both periods 2 and 3. Firms with reserves in area *C* rationally exhaust in period two or earlier. A firm holding reserves in area *D* would prefer to exhaust in period 4 or later.

The next result derives the boundaries for where $q_{i4}^* \geq 0$.

Proposition 4: There exist a set of values $\{R_{i2}, S_{i2}\}$ such that firm *i* optimally ends production in period three.

Proof: See the Appendix.

Propositions 1-4 establish the conditions under firm *i* has a unique equilibrium in which production and exploration are non-negative in periods 2 and possibly period three, given the stocks $\boldsymbol{R}_2$ and $\boldsymbol{S}_2$, and the equilibrium actions of other firms. The next proposition shows the conditions under which a Nash equilibrium among the set of active firms exists and is unique.

Proposition 5: The Nash equilibrium exists and is unique so long as the following conditions hold:

(28) $$P'(Q_t) + q_{it}P''(Q_t) < 0, \qquad i = 1,\ldots,n_2, \text{and } t = 2, 3,$$

(29) $$c_{it}''(q_{it}) - P'(Q_t) > 0, \qquad i = 1,\ldots,n_2, \text{ and } t = 2,3.$$

Proof: Existence (Vives 1999, theorem 2.7). To prove existence, it is necessary to prove that the best-reply functions are strongly decreasing in the output of the other firms. Conditions (28) and (29) ensure that the slope of the best-reply functions $\rho_{i2}(Q_{-i2})$ are strongly decreasing:

$$\rho'_{i2}(Q_{-i2}) \; = \; -\left(\frac{P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3)}{P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3) - (c''_{i2} - P'_2) - \beta(c''_{i3} - P'_3)}\right).$$

Both the numerator and the denominator of the term in brackets are negative, so the whole expression is negative. Therefore, under assumptions (28) and (29), the best-response functions are strictly decreasing. Given this, Vives (1999, theorem 2.7) implies that an equilibrium exists.

Uniqueness. To prove uniqueness, it is necessary to also show that the best-response map $\rho(\cdot) \equiv \{\rho_{12}(Q_{-1}),\ldots,\rho_{n_2 2}(Q_{-n_2})\}$ is a contraction. Vives (1999, theorem 2.8) proves that if the slopes of the best-reply functions are strongly decreasing in the output of the other firms and greater than −1 in value, then a unique equilibrium exists. Note that (28) and (29) imply that

$$0 > P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3) \; > \; P'_2 + q_{i2}P''_2 + \beta(P'_3 + q_{i3}P''_3) - (c''_{i2} - P'_2) - \beta(c''_{i3} - P'_3)$$

Dividing through by −1 times the right-hand-side reveals that $\rho'_{i2}(Q_{-i2}) > -1$. Thus, the condition on the best-response functions is met. This completes the proof.

The conditions (28) and (29) are often called the Hahn conditions, after Hahn (1962). These conditions also imply that the Nash equilibrium in the period two game is stable in the sense of Cournot.

4. STRATEGIC EXPLORATION AND EXTRACTION

Let $V^*_{i2}(\boldsymbol{R}_2,\boldsymbol{S}_2)$ denote the maximized value of problem P2. To obtain strategic effects in exploration, it is necessary for the equilibrium values q^*_{i2}, q^*_{i3}, w^*_{i2}, and w^*_{i3} to depend upon the initial reserves of the other firms. That is, if $q_{it}^* = q_{it}^*(R_{i2}, S_{i2})$ and $w_{it}^* = w_{it}^*(R_{i2}, S_{i2})$ only (i.e., the solutions to the maximization problem P2 depend only own reserves), then there is no strategic effect from production and exploration in period one, since q_{it}^* and w_{it}^* are not affected by the resource stocks of the other firm (Eswaran and Lewis 1986).

We begin by showing the effect on firm i's second period profits of an increase in R_{i2} and S_{i2}. Second period profits for a firm producing in both periods 2 and 3 may be written as

$$(30) \qquad V_{i2}^*(\boldsymbol{R}_2,\boldsymbol{S}_2) \;=\; P(Q_2)q_{i2}^* - c_{i2}(q_{i2}^*) - d_{i2}(w_{i2}^*) + \beta[P(Q_3)q_{i3}^* - c_{i3}(q_{i3}^*) - d_{i3}(w_{i3}^*)].$$

Note that $q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$ for all firms that produce in period three, but for firms for whom (7) does not bind, $w_{i2}^* = w_{i2}(S_{i2})$, while for firms for whom (7) binds, $w_{i2}^* = q_{i2}^* - R_{i2} > w_{i2}(S_{i2})$. Differentiating second period profits with respect to R_{i2} yields

$$(31) \qquad \frac{\partial V_{i2}^*}{\partial R_{i2}} \;=\; \beta(P_3 + P_3' q_{i3}^* - c_{i3}') + (P_2' q_{i2}^* - \beta P_3' q_{i3}^*)\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial R_{i2}}, \qquad \text{(case } A\text{)}$$

if (7) does not bind, and

$$(32) \qquad \frac{\partial V_{i2}^*}{\partial R_{i2}} \;=\; d_{i2}' + \beta(P_3 + P_3' q_{i3}^* - c_{i3}' - d_{i3}') + (P_2' q_{i2}^* - \beta P_3' q_{i3}^*)\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial R_{i2}} \qquad \text{(case } B\text{)}$$

if (7) does bind. Differentiating the second period profit function with respect to S_{i2} yields

$$(33) \qquad \frac{\partial V_{i2}^*}{\partial S_{i2}} \;=\; \beta(P_3 + P_3' q_{i3}^* - c_{i3}' - d_{i3}') + (P_2' q_{i2}^* - \beta P_3' q_{i3}^*)\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial S_{i2}}, \qquad \text{(case } A \text{ \& } B\text{)}$$

for both the case where (7) binds and where it does not bind. The first set of terms on the right-hand-side of (31)-(33) are the direct effects to the firm of having more of that type of stock in period two. By the definitions of the Nash equilibrium given by (18) and (19), these terms are each positive in sign in equilibrium. The term $(P_2' q_{i2}^* - \beta P_3' q_{i3}^*)$ is negative in sign, since this term corresponds to the derivative of firm i's profits with respect to output by firm j, and the firms' goods are substitutes (*cf*. Bulow et al. 1985).

Similarly, the value function for a firm that ends production in period two is

$$V_{i2}^{*}(\boldsymbol{R}_2,\boldsymbol{S}_2) \;=\; P(Q_2)(R_{i2}+S_{i2}) - c_{i2}(R_{i2}+S_{i2}) - d_{i2}(S_{i2}). \tag{34}$$

Differentiating (34) with respect to R_{i2} and S_{i2} yields:

$$\frac{\partial V_{i2}^{*}}{\partial R_{i2}} = P_2 + P_2' q_{i2}^{*} - c_{i2}' + P_2' q_{i2}^{*}\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial R_{i2}}, \qquad \text{(case } C\text{)} \tag{35}$$

$$\frac{\partial V_{i2}^{*}}{\partial S_{i2}} = P_2 + P_2' q_{i2}^{*} - c_{i2}' - d_{i2}' + P_2' q_{i2}^{*}\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial S_{i2}} \qquad \text{(case } C\text{).} \tag{36}$$

By the chain rule, the sign of the effects of R_{i2} and S_{i2} on q_{j2}^{*} in (31)-(33) and (35)-(36) can be written as

$$\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial R_{i2}} = \left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}}\right)\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}} \qquad \text{and} \qquad \sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial S_{i2}} = \left(\frac{\partial q_{i2}^{*}}{\partial S_{i2}}\right)\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}}. \tag{37}$$

The sign of these expressions depend on the slopes of the best-response functions of all other firms to firm *i*'s output level. Given equations (28) and (29), the goods are strategic substitutes in the sense of Bulow et al. (1985). However, we need these to be negative in net given the interactions among the set of all other firms.[11] Proposition 6 shows that this is so.

Proposition 6: The sum of the $\partial q_{j2}^{*}/\partial q_{i2}$ are negative for firms that produce in period two and three, and zero for firms that end production in period two.

Proof: Write the total differential of the j^{th} firm's first order condition on the choice of q_{j2}^{*} as

[11] The best response functions $q_{i2}^{*} = \rho_i(Q_{-i2})$ describe how firm *i* responds to changes in the output of all other firms. To see how all other firms simultaneously respond to a change in firm *i*'s output, we need to solve the system of equations $H_{-i}\boldsymbol{dq}_{-i2} = bdq_{i2}$ to obtain $\boldsymbol{dq}_{-i2} = H_{-i}^{-1}bdq_{i2}$, where $\boldsymbol{dq}_{-i2} = \{dq_{12},\ldots,dq_{i-12},dq_{i+12},\ldots,dq_{n2}\}$ is the vector of dq_{j2} for $j\neq i$, H_{-i} is the Jacobian matrix for the first-order conditions for all firms other than firm *i* with diagonal elements a_j equal to the second order conditions on q_{j2} and off diagonal elements b_j in row *j* (where these are defined in the text below), and $b = \{b_1,\ldots,b_{i-1},b_{i+1},\ldots,b_{n2}\}$ is the vector of the cross-effects on marginal profits.

(38) $a_j dq^*_{j2} + b_j \Sigma_{k \neq i,j}\, dq^*_{k2} \;=\; -b_j dq_{i2},\qquad j \neq i,$

where $a_j \equiv 2P'_2 + P''_2 q^*_{j2} - c''_{j2} + \beta(2P'_3 + P''_3 q^*_{j3} - c''_{j3}) < 0$ for a firm for whom (7) is not binding and $a_j \equiv 2P'_2 + P''_2 q^*_{j2} - c''_{j2} - d''_{j2} + \beta(2P'_3 + P''_3 q^*_{j3} - c''_{j3} - d''_{j3}) < 0$ for a firm for whom (7) is binding, and $b_j = P'_2 + P''_2 q^*_{j2} + \beta(P'_3 + P''_3 q^*_{j3}) < 0$ for all firms that continue to produce in period three. These expressions are each negative by the Hahn conditions. We can rewrite (38) as

$$dq^*_{j2} + \left(\frac{b_j}{a_j - b_j}\right) dQ_{-i2} \;=\; -\left(\frac{b_j}{a_j - b_j}\right) dq_{i2}.$$

Summing over all $j \neq i$ and solving for how the aggregate output by other firms changes as q_{i2} increases yields

$$\frac{dQ_{-i2}}{dq_{i2}} \;=\; -\left(1 + \sum_{j \neq i} \frac{b_j}{a_j - b_j}\right)^{-1} \sum_{j \neq i} \frac{b_j}{a_j - b_j}.$$

Thus

$$\frac{\partial q^*_{j2}}{\partial q_{i2}} \;=\; -\left(\frac{b_j}{a_j - b_j}\right)\left(1 + \sum_{k \neq i} \frac{b_k}{a_k - b_k}\right)^{-1} \;<\; 0, \tag{39}$$

since $b_j/(a_j - b_j) > 0$ for all j. This completes the proof for those firms that produce into period three. For firms that end production in period two, $q^*_{j2} = R_{j2} + S_{j2}$. Thus, these firms do not respond at all to changes in q_{i2}. This completes the proof.

Next, for each firm that produces in periods 2 or 3, we show that its own second period output is increasing in its own second period proven reserves.

Proposition 7: Second period output is increasing in second period proven reserves for firms that produce in periods 2 and 3.

Proof: Write the total differential of a firm that produces in both periods 2 and 3 first-order condition in its own output q_{i2}^* as

(40) $$a_i dq_{i2}^* + b_i \Sigma_{j \neq i}\, dq_{j2}^* \;=\; -c_i dR_{i2} - e_i dS_{i2},$$

where a_i and b_i are defined as above, and where $c_i = e_i \equiv -\beta(2P_3' + P_3''q_{i3}^* - c_{i3}'') > 0$ for firms for whom (7) is not binding, and $c_i = d_{i2}'' - \beta(2P_3' + P_3''q_{i3}^* - c_{i3}'' - d_{i3}'') > 0$ and $e_i = -\beta(2P_3' + P_3''q_{i3}^* - c_{i3}'' - d_{i3}'') > 0$ for a firm for whom (7) is binding. For the case where R_{i2} changes, (40) implies

(41) $$\frac{\partial q_{i2}^*}{\partial R_{i2}} = \frac{-c_i}{\left(a_i + b_i \sum_{j\neq i} \frac{\partial q_{j2}^*}{\partial q_{i2}}\right)} = \left(\frac{c_i}{-(a_i - b_i)}\right)\left(\frac{1 + \sum_{j\neq i} \frac{b_j}{a_j - b_j}}{\frac{a_i}{a_i - b_i} + \sum_{j\neq i} \frac{b_j}{a_j - b_j}}\right) \equiv c_i \Gamma_i > 0,$$

where Γ_i is $-(a_i - b_i)^{-1}$ times the second expression in brackets in the second equality. Γ_i is positive since $a_i - b_i < 0$ and both $a_i/(a_i - b_i) > 0$ and $b_i/(a_i - b_i) > 0$ for all i. Thus, $\partial q_{i2}^*/\partial R_{i2} = c_i\Gamma_i > 0$ and by an equivalent process, it can be shown that $\partial q_{i2}^*/\partial S_{i2} = e_i\Gamma_i > 0$. This completes the proof for firms that produce in both periods 2 and 3.

For firms that produce in only period two, $q_{i2}^* = R_{i2} + S_{i2}$, so that $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2} = 1$. This completes the proof.

Note that when (7) is not binding, $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2}$, and when (7) is binding, $\partial q_{i2}^*/\partial R_{i2} - \partial q_{i2}^*/\partial S_{i2} = (c_i - e_i)\Gamma_i = d_{i2}''\Gamma_i > 0$. These results affect whether or not exploration gives the firm a strategic advantage, since an increase in first period exploration, w_{i1}, increases R_{i2} and decreases S_{i2}.

Next, we state a sufficient condition for $\partial q_{i2}^*/\partial R_{i2} < 1$ and $\partial q_{i2}^*/\partial S_{i2} < 1$ for firms that produce in periods 2 and 3:

Corollary to Proposition 7: For firms that produce in both periods 2 and 3, a sufficient condition for $\partial q_{i2}^*/\partial R_{i2} < 1$ is that $a_i - b_i + c_i < 0$, and a sufficient condition for $\partial q_{i2}^*/\partial S_{i2} < 1$ is that $a_i - b_i + e_i < 0$.

Proof: We shall show that $a_i - b_i + c_i < 0$ is sufficient for $\partial q_{i2}^*/\partial R_{i2} < 1$. Observe that

$$a_i + c_i + (a_i - b_i + c_i)\sum_{j\neq i} \frac{b_j}{a_j - b_j} < 0$$

if $a_i - b_i + c_i < 0$, since $a_i + c_i < 0$ and $\Sigma_{j\neq i} b_j/(a_j - b_j) > 0$. This expression can be rearranged to show that the right-hand-side of (41) is less than 1. This completes the proof.

The condition that $a_i - b_i + c_i < 0$ can be rewritten as

$$P_2' - c_{i2}'' - d_{i2}'' < \beta(P_3' + P_3''q_{j3}^*).$$

The left-hand-side of this inequality is the Hahn condition that demand intersect the marginal cost from above. The right-hand-side is the effect other firms impose upon the firm by increasing output. Observe that with linear demand and exponential cost functions, this condition is satisfied, although in general it may not be satisfied. However, this condition is apparently satisfied in the world oil market, since the data in figure 3 supports the Loury-Polasky hypothesis, which in the strategic model, requires the conditions of the Corollary to hold.

The terms in (31)-(33) and (35)-(36) involving the summations from (37) are the strategic effects of holding higher level of reserves. In an open loop equilibrium, where the other firms observe R_{i1} and S_{i1}, but not R_{i2} and S_{i2}, we see that the effect of holding stocks in the second period is positive. Since the terms involving the summations from (37) are together positive in sign, we see that in the subgame perfect equilibrium, firms have an incentive to hold greater levels of both types of reserves than in the open loop equilibrium.

Now we are ready to consider the problem faced by firm i in period one. The objective of a firm in period one is to choose output q_{i1} and exploration w_{i1} to maximize

P1 $$\max_{\{q_{i1}\, w_{i1}\}} \; V_{i1} = P(Q_1)q_{i1} - c_{i1}(q_{i1}) - d_{i1}(w_{i1}) + \beta V_{i2}^{*}(\boldsymbol{R}_2,\boldsymbol{S}_2), \qquad i = 1,\ldots,n_1,$$

where the value function $V_{i2}^{*}(\boldsymbol{R}_2,\boldsymbol{S}_2)$ is given by (30) or (34), depending on whether or not the firm produces in period three. Firm i's choices are subject to the constraints

(42) $w_{i1} \le S_{i1},$

(43) $w_{i1} \ge 0,$

(44) $q_{i1} \le R_{i1} + w_{i1},$

which are analogous to the constraints (5)-(7), respectively.

Let γ_i, δ_i and κ_i denote the value of Lagrange multiplier on the constraints (42)-(44), respectively. By the envelope theorem, the solution to P1 for a firm that produces in periods 2 and 3 when (7) is not binding must satisfy (42)-(44) and:

(45) $$\frac{\partial V_{i1}}{\partial q_{i1}} = P(Q_1) + P'(Q_1)q_{i1}^{*} - c_{i1}'(q_{i1}^{*}) - \kappa_i - \beta^2[P(Q_3) + P'(Q_3)q_{i3}^{*} - c_{i3}'(q_{i3}^{*})]$$
$$- \beta\big[P'(Q_2)q_{i2}^{*} - \beta P'(Q_3)q_{i3}^{*}\big]\left(\sum_{j\ne i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}}\right) = 0, \qquad i = 1,\ldots,m_1, \text{ (case } A\text{)}$$

(46) $$\frac{\partial V_{i1}}{\partial w_{i1}} = -d_{i1}'(w_{i1}^{*}) + \kappa_i + \delta_i - \gamma_i - \beta d_{i2}'(w_{i2}^{*})$$
$$+ \beta\big[P'(Q_2)q_{i2}^{*} - \beta P'(Q_3)q_{i3}^{*}\big]\left(\sum_{j\ne i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}} - \frac{\partial q_{i2}^{*}}{\partial S_{i2}}\right) = 0, \quad i = 1,\ldots,n_1 \text{ (case } A\text{)}.$$

The strategic effects appear in the terms of the second lines in (45) and (46), which have been written using (31)-(33). These are strategic effects because firm i chooses the stocks it takes into period two, knowing the effect this has upon the exploration and output choices of other firms in subsequent periods. Absent these effects, the equilibrium is identical to the open loop equilibrium (Eswaran and Lewis 1986), which solves the expressions on the first lines of (45) and (46) set equal to zero.

An immediate result, which follows from Proposition 6, is the following:

Proposition 8: Firms that hold reserves in sufficient quantities that they will produce in both periods 2 and 3 and for whom the constraint (7) is not binding (class *A* firms) have a strategic reason to restrict output, but do not have a strategic reason to increase exploration.

Proof: When (7) is not binding, (41) implies that $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2}$, so the strategic effect vanishes. Thus, there is no strategic effect from exploration. To see that the strategic effect decreases first period production, rewrite (45) as

$$(45') \qquad P(Q_1) + P'(Q_1)q_{i1}^* - c'_{i1}(q_{i1}^*) - \kappa_i = \beta[P(Q_2) + P'(Q_2)q_{i2}^* - c'_{i2}(q_{i2}^*)] + \beta\left[P'(Q_2)q_{i2}^* - \beta P'(Q_3)q_{i3}^*\right]\left(\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^*}{\partial R_{i2}}\right).$$

The expression on the left-hand-side is the marginal profit from period one production. The expression on the right hand side is the marginal profit from second period production plus (the term on the second line) the strategic effect of holding higher reserves in period three. As the strategic effect is in net positive in sign, the firm has a greater incentive to withhold production in the first period relative to the open loop equilibrium. This completes the proof.

Next, consider the equivalent conditions for a firm for whom the constraint (7) is binding along the equilibrium path. The equivalent first-order conditions to (45) and (46) are

$$(47) \qquad P(Q_1) + P'(Q_1)q_{i1}^* - c'_{i1}(q_{i1}^*) - \kappa_i = \beta d'_{i2} + \beta[P_2 + P'_2 q_{i2}^* - c'_{i2} - d'_{i2}] + \beta\left[P'(Q_2)q_{i2}^* - \beta P'(Q_3)q_{i3}^*\right]\left(\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^*}{\partial R_{i2}}\right), \qquad i = 1,\ldots,m_1 \text{ (case } B\text{)},$$

$$(48) \qquad d'_{i1}(w_{i1}^*) - \kappa_i - \delta_i + \gamma_i = \beta d'_{i2} + \beta\left[P'(Q_2)q_{i2}^* - \beta P'(Q_3)q_{i3}^*\right]\left(\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^*}{\partial R_{i2}} - \frac{\partial q_{i2}^*}{\partial S_{i2}}\right), \quad i = 1,\ldots,n_1 \text{ (case } B\text{)}.$$

The next proposition summarizes the strategic effects for this type of firm:

Proposition 9: Firms that hold reserves in sufficient quantities that they will produce in both periods 2 and 3, but for whom the constraint (7) is binding (class *B* firms) have both a strategic reason to restrict output and a strategic reason to increase exploration.

Proof: The strategic interaction terms appear on the second lines of (47) and (48). Both are positive in sign by Propositions 6 and 7. This completes the proof.

Thus, firms whose holdings of reserves are sufficient to get to period three but with positive quantities of proven reserves are able to exert strategic pressure on those firms with large enough reserves that they still have proven reserves at the end of period three.

Next, consider a firm that exhausts its resource stock in period two. For this type of firm, the equivalent conditions for maximizing P1 are

$$(49) \qquad P(Q_1) + P'(Q_1)q_{i1}^* - c'_{i1}(q_{i1}^*) - \kappa_i = \beta[P_2 + P'_2 q_{i2}^* - c'_{i2}] + \beta P'(Q_2)q_{i2}^*\left(\sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^*}{\partial R_{i2}}\right),$$

$$(50) \qquad d'_{i1}(w_{i1}^*) - \kappa_i - \delta_i + \gamma_i = \beta d'_{i2} + \beta P'(Q_2)q_{i2}^*\left(\sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^*}{\partial R_{i2}} - \frac{\partial q_{i2}^*}{\partial S_{i2}}\right), \qquad \text{(case } C\text{)}.$$

Proposition 10: For a firm with sufficient reserves to produce in period two, but insufficient reserves to produce in period three, there will be a strategic effect from withholding production, but not one from increasing exploration.

Proof: Recall that $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2} = 1$ for firm that exhausts in period two, since $q_{i2}^* = R_{i2} + S_{i2}$. Thus, the strategic effect vanishes in the exploration equation, but remains in the production equation. This completes the proof.

Propositions 8-10 suggest that only firms that exhaust their proven reserves in period two have a strategic incentive to explore for oil. Firms with sufficient reserves to have proven reserves at the beginning of period three already have sufficient reserves to have a credible commitment that they will produce a large quantity in period three. Thus, they do not gain anything by exploring beyond the level that would occur in the open loop equilibrium.

Now that we have seen the strategic effects, there remains but one task. That is to characterize the equilibrium in period one. We begin our analysis of the equilibrium in period one by ignoring the strategic effects and assuming that none of the constraints (42)-(44) binds. Then the choice of production given by (45), (47) or (49) says that marginal profits from extraction in period one are equated with the discounted value of additional proven reserves in period three. Thus, this is again a simple Hotelling result which implies that discounted marginal profits are equated across periods. Equations (46), (48) and (50) give a similar Hotelling result that discounted marginal costs of exploration are equal across periods. Obviously, the strategic effects alter the interpretation of these results, as will having any of the constraints (42)-(44) bind.

Next, we characterize the equilibrium in period one by showing which constraints can be binding and which cannot.

COROLLARY TO PROPOSITION 1: If $S_{i1} > 0$ and firm *i* produces for two or more periods, it will explore in every period.

Proof: The proof appears in the appendix.

Next, we show that if the constraint (44) is binding, so that firm *i*'s proven reserves in period two are zero, then firm *i* will not have positive proven reserves at the end of any subsequent period.

PROPOSITION 11: If a firm extracts all of its proven reserves in period one, it will not subsequently hold positive quantities of proven reserves.

Proof: By assumption, $R_{i2} = 0$. Now, suppose that the conclusion does not follow. Then it must be that $q_{i2}^{*} < w_{i2}^{*}$, if the firm continues to produce to period three. (If the firm does not continue to produce in period three, then all reserves are exhausted in period two, which proves the proposition.) Thus $\phi_i = 0$, since (7) is not binding. Since the feasibility constraint (4) must bind, it requires that $q_{i3}^{*} > w_{i3}^{*}$. However, $\phi_i = 0$ implies that w_{i2}^{*} solves (15), so that $w_{i3}^{*} > w_{i2}^{*}$. Therefore $q_{i2}^{*} < w_{i2}^{*} < w_{i3}^{*} < q_{i3}^{*}$. However $\phi_i = 0$ also implies that q_{i2}^{*} solves (17), so that $q_{i2}^{*} > q_{i3}^{*}$. This is a contradiction, which completes the proof.

These propositions eliminate all but three possible combinations of exploration activities for the $n_1 - s_1 - s_2$ firms that produce in all three periods.[12] We can conclude that so long as initial unproven reserves are positive, the firm explores in every subsequent period. Furthermore, for each period in which firm *i* takes some proven reserves into the next period, it extracts only from the lower cost proven reserves. Lastly, if proven reserves are exhausted prior to unproven reserves, then the firm will not rebuild these proven reserves in any subsequent period.

5. Convergence in Reserves?

This paper has shown that in an oligopolistic industry, exploration is likely to be higher than suggested by simple Cournot-Nash models. The reason for this is that an oligopolistic firm takes into account the strategic effect sinking exploration costs has on its rivals. Because of the homogeneous good being produced by the oligopolists, the goods are strategic substitutes, which means that an increase in exploration today lowers future production by rivals.

Perhaps the most interesting conclusion from this study is that smaller firms – or at least

[12] There are also s_2 firms which only hold sufficient reserves to produce in period two (those who take reserves equivalent to the area *C* in Figure 3), and there are s_1 firms whose reserve holdings are insufficient to even produce in period two. We show below that these firms' choices cannot be influenced by the actions of the remaining firms, but those who take reserves into period two can affect the behavior of firms who continue to produce into period three.

those with smaller proven reserves – are those that will do more exploration, all else equal. As we argued above, the intuition behind this result is that firms with large proven reserves already have a credible commitment to produce large quantities. Also, because these firms are large producers they gain less from a reduction in costs because a large producer faces a larger reduction in price when it expands its output (e.g., Nordhaus 1969). Thus, smaller firms are those who do the most exploration. Furthermore, a firm with large proven reserves already has sufficient low cost reserves on hand to credibly increase its output.

The data in figure 4 suggested that smaller firms tended to do more exploration. A simple test of this hypothesis is to regress the percentage change in reserves for each country against its initial reserves. The resulting "convergence" equation is:

$$\frac{ln(R_{T,i}) - ln(R_{0,i})}{T_i} = \underset{(0.51)}{4.08} - \underset{(0.24)}{1.11}\, ln(R_{0,i}) \qquad i=1,\ldots,99, \quad \text{Adj.-}R^2 = 0.18.$$

The standard errors are in parentheses. T_i is the number of years each country is observed in the data, and $ln(R_{T,i})$ and $ln(R_{0,i})$ are the log of ending and beginning reserves, respectively. This regression, coupled with the results reported in figures 2 and 4, supports the hypothesis that smaller countries do indeed explore more relative to the larger countries.

Two final comments are in order. First, the correlations depicted in figures 3 and 4 show vastly different levels of precision. The standard error was around four percent of the magnitude of the slope coefficient for the production to reserves correlation while the standard error was nearer to twenty percent of the magnitude of the slope coefficient in the growth of reserves to initial reserves regression. We have not attempted to explain this variation. However, recent papers by Bohn and Deacon (2000) and by Sachs and Warner (2001) suggest that these differences may be attributable to institutional quality differences across countries. This would show up more in exploration data than in production data simply because firms considering exploration have much longer planning horizons than firms making production decisions.

Finally, we appear to be entering another cycle where there is much concern that the world is running out of oil. The so-called Hubbert date, named after the geologist who correctly predicted that oil production in the United States would begin to decline in the early 1970s, when world oil production is expected to begin to decline has been placed by some as to be occurring in the next year or two.[13] Our model is both consistent and inconsistent with this view. If firms are over-exploring relative to what they would do if they ignored the strategic effects they have upon their rivals' output, then the current level of proved reserves may be higher relative to unproved reserves than would occur if firms ignored the strategic effects of exploration. However, this incentive only occurs with smaller firms. Larger producers do not have this incentive. Thus, there may still be large discoveries to be found in exactly the same countries as where large proven reserves currently exist. Those producers simply do not have an incentive to explore at this time.

MATHEMATICAL APPENDIX

Proof of Proposition 1

This proposition is proven by the following three lemmas. In each case, it is assumed that the firm produces in period 3, which implies that $\alpha_i = 0$.

LEMMA 1: With positive quantities of the unproven reserves, if there is zero exploration in some period, it will be in period two, not period three.

Proof: Suppose not. Suppose that $w_{i2} > 0$ and $\theta_{i3} > 0$. Then, $\theta_{i2} = 0$ and $w_{i3} = 0$. From (10) and (11), we

[13] See Campbell and Laherrere (1998) for a recent example of this view.

obtain that

$$\text{(A.1)} \qquad d'_{i2}(S_{i2}) - \phi_i = \lambda_i - \mu_i < \beta d'_{i3}(0).$$

Since the firm is assumed to extract in the third period, it is not possible that $w_{i2} = S_{i2} > 0$ and $\phi_i > 0$ both occur. Thus, let $\phi_i = 0$. Then this equation implies that $d'_{i2}(S_{i2}) = -\theta_{i3} < 0$, which is a contradiction, since $d'_{i2}(w_{i2}) > 0$ for $w_{i2} > 0$.

Next, we prove that it is not possible for $\phi_i > 0$ and $\theta_{i2} > 0$ simultaneously.

LEMMA 2: If firm *i* extracts all of its proven reserves in period two, then it must also explore in period two.

Proof: Suppose not. Suppose that $\phi_i > 0$ and that $\theta_{i2} > 0$. Then $\theta_{i2} > 0$ implies that $w_{i2} = 0$, so that $w_{i3} = S_{i2}$. Thus, (10) and (11) imply $\beta d'_{i3}(S_{i2}) + \phi_i + \theta_{i2} = 0$, since $d'_{i2}(0) = 0$. This is contradiction since all three terms on the left side of this equation are positive.

LEMMA 3: It is not possible for exploration to be zero in period two when the period two extraction constraint (6) is not binding.

Proof: Suppose not. Suppose that $\theta_{i2} > 0$ and $q_{i2} < R_{i2} + w_{i2}$. These imply that $w_{i2} = \phi_i = 0$. Thus $w_{i3} = S_{i2}$ by (5), and (11) implies that $\lambda_i - \mu_i = \beta d'_{i3}(S_{i2})$, which is positive in sign. Thus, (10) implies that $\theta_{i2} = -\beta d'_{i3}(S_{i2}) < 0$, which contradicts $\theta_{i2} > 0$.

Lemma 3 completes the proof of Proposition 1.

<u>Proof of Proposition 4</u>

Firm *i* rationally ends production in period three only if

$$\text{(A2)} \qquad \tau_i(R_{i2}, S_{i2}) = \pi_1^{i3}(q_{i3}^*, Q_{-i2}) - d'_{i3}(w_{i3}^*) - \beta\, \pi_1^{i4}(0, 0) \geq 0.$$

(i) Suppose that $q_{i2}^* < R_{i2} + w_{i2}^*$. Then $q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$, and $w_{i3}^* = S_{i2} - w_{i2}^*$. Thus, let

$$\text{(A.3)} \qquad \tau_{i1}(R_{i2}, S_{i2}) \equiv \pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i2}) - d'_{i3}(S_{i2} - w_{i2}^*) - \beta\pi_1^{i4}(0, 0).$$

When $S_{i2} = 0$, the value of $R_{i2} = \hat{R}$ such that $\tau_{i1}(\hat{R}, 0) = 0$ must satisfy $\pi_1^{i3}(\hat{R} - q_{i3}^*, Q_{-i3}) = \beta\pi_1^{i4}(0, 0)$. It can be shown (*cf.* (41)) that $0 < \partial q_{i2}^*/\partial R_{i2} < 1$. Thus, $\hat{R} - q_{i2}^*$ lies between zero and $\hat{R}$. Let $\bar{R}$ solve $\pi_1^{i2}(\bar{R}, Q_{-i2}) \equiv \beta\pi_1^{i3}(0,Q_{-i3})$, which is the boundary given in Proposition 3 for ending production in period 2. Since $\hat{R} - q_{i2}^*$ is strictly positive, it follows that $\hat{R} > \bar{R}$. Thus, in the region where $S_{i2} = 0$, there exists a set of values of R_{i2} such that firm *i* wishes to exhaust in period three and follow the strategy outlined in proposition 2. It can also be shown that along the locus of points where $\tau_{i1}(R_{i2}, S_{i2}) = 0$, that

(A.4) $$\left.\frac{dR_{i2}}{dS_{i2}}\right|_{\tau_{i1}(R_{i2},S_{i2}) = 0} = -\frac{(1 - \partial q_{i2}^*/\partial S_{i2})\pi_1^{i3} - (1 - \partial w_{i2}^*/\partial S_{i2})d_{i3}''}{(1 - \partial q_{i2}^*/\partial R_{i2})\pi_1^{i3}} < 0,$$

Since $0 < \partial w_{i2}^*/\partial S_{i2} < 1$, and $0 < \partial q_{i2}^*/\partial S_{i2} = \partial_{i2}^*/\partial R_{i2} < 1$.

(ii) Next, consider the case where $q_{i2}^* = R_{i2} + w_{i2}^*$. Then $w_{i3}^* = q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$. In this case, let

(A.5) $$\tau_{i2}(R_{i2}, S_{i2}) \equiv \pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3}) - d_{i3}'(R_{i2} + S_{i2} - q_{i2}^*) - \beta\pi_1^{i4}(0, 0),$$

where q_{i2}^* solves $\psi_i(q_{i2}^*) = 0$. In this case, the $\tau_{i2}(R_{i2}, S_{i2})$ loci is again downward sloping:

(A.6) $$\left.\frac{dR_{i2}}{dS_{i2}}\right|_{\tau_{i2}(R_{i2},S_{i2}) = 0} = -\frac{1 - \partial q_{i2}^*/\partial S_{i2}}{1 - \partial q_{i2}^*/\partial R_{i2}} < 0,$$

since $0 < \partial q_{i2}^*/\partial S_{i2} < \partial q_{i2}^*/\partial R_{i2} < 1$.

This implies that there exist values $\{R_{i2}, S_{i2}\}$ such that firm *i* wishes to produce in period three but not in period 4. When $R_{i2} = 0$, the corresponding value of $S_{i2} = \hat{S}$ such that $\tau_{i2}(0, \hat{S}) = 0$ must satisfy

(A.7) $$\pi_1^{i3}(\hat{S} - q_{i2}^*, Q_{-i3}) - d_{i3}'(\hat{S} - q_{i2}^*) = \beta\pi_1^{i4}(0, 0).$$

Let $\bar{S}$ solve $\pi_1^{i2}(\bar{S}, Q_{-i2}) \equiv \beta\pi_1^{i3}(0,Q_{-i3})$, which is the boundary given in Proposition 3 for ending production in period 2, Since $0 < \partial q_{i2}^*/\partial S_{i2} < 1$, $\hat{S} - q_{i2}^*$ is strictly positive. This implies that in the region where $R_{i2} = 0$, that $\hat{S} > \bar{S}$, which completes the proof.

Proof of the Corollary to Proposition 1

The proof follows from lemmas 4 and 5:

LEMMA 4: Zero exploration activity in the first period cannot be an equilibrium, unless $S_{i1} = 0$.

Proof: Assume not. Suppose $w_{i1} = 0$ which implies that $\delta_i > 0$. Then $S_{i1} > 0$ by assumption of positive initial unproven reserves and so $\gamma_i = 0$. Equation (46) and (48) become

$$\text{(A.8)} \qquad \kappa_i + \delta_i + \beta \frac{\partial V_{i2}^{*}}{\partial w_{i1}} = 0, \qquad i = 1, 2.$$

In this case, when $w_{i1} = 0$ the partial derivatives with respect to w_{i1} equal zero and we have $\kappa_i + \delta_i = 0$. This is the contradiction since the sum of positive values of Lagrange multipliers cannot result in zero. This lemma is independent of whether or not the constraints (7) or (44) bind.

LEMMA 5: If each firm lasts three periods, no firm exhausts all unproven reserves in the first period.

Proof: Suppose not. Suppose $w_{i1} = S_{i1} > 0$, $\delta_i = 0$, and $\gamma_i > 0$. It must follow that $\kappa_i = 0$ and $\phi_i = 0$, otherwise firm i has no extraction from proven reserves in later periods. Using Proposition 8, (46) becomes

$$\text{(A.9)} \qquad -d_i'(S_{i1}) + \beta(\lambda_i - \mu_i) - \gamma_i = 0.$$

From (10) and (11), we see that $\lambda_i - \mu_i = -\theta_{i2} = -\theta_{i3}$. Thus, $-(\beta\theta_i + \gamma_i) = d_{i1}'(S_{i1}) > 0$, which is a contradiction.

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