Coordination Cascades:
Sequential Choice in the Presence of a Network Externality

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Abstract

In the network externality literature, little, if any attention has been paid to the process through which consumers coordinate their adoption decisions. The primary objective of this paper is to discover how effectively rational individuals manage to coordinate their choices in a sequential choice framework. Since individuals make their choices with minimal information in this setting, perfect coordination will rarely be achieved, and it is therefore of some interest to discern both the extent to which coordination may be achieved, and the expected cost of the failure to achieve perfect coordination. We discover that when it counts, that is when the network externality is large, a substantial amount of coordination is achieved, and although perfect coordination is never guaranteed, expected relative efficiency is large.

Keywords: Coordination Cascades, Network Externalities, Social Networks, Sequential Choice

JEL Classification: L13

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1 Introduction

There is a large literature on network externalities.[1] In this literature, the existence of a network externality implies that the realized network benefit that an individual receives is dependent upon the number of other individuals who make the same choice. As a result, when making her adoption decision an individual must form expectations concerning the number of individuals who will eventually choose the same good. Despite the large literature on network externalities, relatively little work has focused on the expectations formation problem that is at the core of the individual's choice problem in the presence of a network externality. Typically, the problem is addressed with a simplifying assumption, or it is ignored altogether. For example, Katz and Shapiro (1985) resolve the problem by adopting the assumption that consumer expectations are fulfilled in equilibrium. Of course, models with network externalities have multiple equilibria, which raises yet another expectations formation question, even if it is supposed that expectations are fulfilled in equilibrium: how is it that consumers manage to coordinate their expectations on one of the possible equilibria? That is, network externalities raise an equilibrium selection problem. To achieve any of the equilibria, the expectations of all individuals must somehow be coordinated. But if choices are made independently, there is nothing to assure that coordination will in fact, be achieved. These problems are well known, and various authors have suggested a variety of approaches to them (see for example, Farrell and Katz (1998)), yet they remain unresolved. This is surprising given how important expectations are when dealing with network externalities. As Farrell and Klemperer (2004) point out, "how adopters form expectations and coordinate their choices dramatically affects the performance of competition among networks."

Notice also that when the process by which expectations are formed is swept

[1] See http://www.stern.nyu.edu/networks/site.html. See especially, Rohlfs (1974), Katz and Shapiro (1985), Farrell and Saloner (1985), Church and Gandal (1992), Katz and Shapiro (1994), and Economides (1996).

away, so too is the ability to address a variety of interesting questions. Is it easier to achieve coordination as the total number of consumers increases? If the value of the network benefit is small, is coordination more difficult to achieve? Is it easier to coordinate in an environment where tastes are similar? Are the expected costs of the failure to achieve perfect coordination large? More generally, what facilitates and what inhibits coordination? Our purpose in this paper is to explore these issues in a sequential choice framework.

We approach the coordination problem in a framework that is suggested by the literature on information cascades.[2] In our model, choices are made sequentially, and in making their choices individuals know their own preferences, the choices of people who precede them in the choice making sequence, and very little else. Clearly, in this setting perfect coordination will rarely be achieved, and it is therefore of some interest to discern both the extent to which coordination may be achieved, and more particularly, the expected cost of the failure to achieve perfect coordination, since this magnitude would seem to have a direct bearing on the question of whether or not unfettered markets can effectively solve the coordination problem associated with a network externality. To preview results, it turns out that if the network externality in our model is sufficiently strong, individuals manage to achieve substantial coordination.

The particular model that we use is a special case of the model of Katz and Shapiro (1985).[3] There are two goods, and two types of people. One type has a private preference for one of the goods and the other has a private preference for the other good. However, there is a network externality that potentially overwhelms these private preferences. We restrict attention to parameter values such that there

[2]For a further discussion on information cascades see Bikhchandi et al (1992, 1998).

[3]While the discussion in this paper focuses on issues that arise in the context of industrial organization, the analysis is equally applicable to the literature on network externalities in the context of social interaction. See Leibenstein (1950), Akerlof (1980,1997), Becker (1991), Banerjee (1992), Glaeser and Scheinkman (2000), Eaton,Pendakur and Reed (2001) and Durlauf and Young (2001).

are two equilibria of the simultaneous choice game, in both equilibria all individuals choose the same good, and both equilibria are Pareto-optimal. We examine both the simultaneous move game, and the game that arises when decisions are made sequentially, and compare the equilibrium of the sequential choice game with the equilibrium of the corresponding simultaneous choice game.

Choi (1997) explores a technology adoption problem with network externalities in a sequential choice setting. There are two technologies, A and B, and initially the effectiveness of both is unknown. But once the first adopter has chosen a technology, say technology A, its effectiveness is revealed to all. The second adopter then faces the problem of whether to choose the untried technology B, or to follow the first adopter's lead and choose technology A. Because there are network effects, the second adopter must ask herself what those who follow her will do. If she chooses B, and it turns out to be more effective than A, those who follow will also choose B and the second adopter will have a large network and the more effective technology. But if she chooses B and it turns out to be less effective than A, those who follow will adopt A, and the second mover will be stranded with no network. Hence, if the value of a network is large, the expected effectiveness of technology B must be significantly greater than the known effectiveness of technology A to induce the second adopter to choose technology B. In that choices are sequential, this model is similar to ours. However, the nature of the information problem is quite different. In our model, individuals use the choices of those who precede them in the choice sequence to infer something about the relative proportions of types of individual in the population, while in Choi's model the choices of predecessors resolve uncertainty about the technologies. As will become clear, although the models are similar in one respect, the analytical approaches are quite different, as are the results.

2 The Model

In our model there are two goods, *good 1* and *good 2*, there are two types of individual, *type A* and *type B*, there are many individuals of each type, and individuals choose either good 1 or good 2. Goods have a purely private value that is type dependent, and a network or social value that is driven by a numbers externality.

There are N_A individuals of type A, N_B of type B, and the total number of individuals is $N \equiv N_A + N_B$. For a person of type i, the private value of good j is z_{ij}. We assume that $z_{A1} > z_{A2}$ and that $z_{B2} > z_{B1}$, so that type A individuals have a private preference for good 1, and type B a private preference for good 2. For simplicity we assume that $z_{A1} > 0$ and $z_{B2} > 0$. For type A individuals the magnitude of the *private preference differential* is $D_A \equiv z_{A1} - z_{A2}$, and for type B individuals it is $D_B \equiv z_{B2} - z_{B1}$. In most of what we do in this paper, these differentials are identical.

Network externalities are captured by a parameter $S > 0$ that we call *potential network value*. If some person chooses good j, the *realized network value* for that person is $S \cdot \pi_j$, where π_j is the proportion of other people who also choose good j. In other words, the potential network value is realized by an individual only to the extent that the individual's consumption decision is coordinated with the consumption decisions of others.

Individuals choose either good 1 or good 2, but not both. Although some interesting pricing issues arise in this model, we do not address them. Hence there is no need to explicitly introduce prices. Instead, interpret z_{ij} as the private value to individual i of good j net of the price of the good.

If $D_i > S$, then private consumption values drive the choice of an individual of type i. Consider, for example, a type A individual: $D_A > S$ implies that $z_{A1} > z_{A2}+S$ so that the individual always prefers good 1 to good 2. Note that when both $D_A > S$

and $D_B > S$, there is a unique equilibrium in which type A individuals choose good 1 and type B individuals choose good 2, and that this equilibrium is Pareto-optimal. In this case, the network externality raises no interesting issues. Accordingly, we assume that $D_A < S$ and $D_B < S$.

If we now imagine that choices are made simultaneously, there are two corner equilibria in which everyone chooses either good 1 or good 2. To see this, consider an individual of type A faced with a situation in which she anticipates that everyone else will choose good 2. Since $D_A < S$, it is the case that $S + z_{A2} > z_{A1}$, and the network effect leads the type A individual to choose good 2 in preference to her privately preferred good, good 1. Since type B individuals clearly prefer good 2 when they anticipate that everyone else will choose good 2, we have a corner equilibrium in which everyone chooses good 2.

Notice that both corner equilibria are Pareto-optimal. Consider, for example, the corner equilibrium in which everyone chooses good 2. In this equilibrium individuals of type B are better off than they would be in any other situation, since they consume the good that they privately prefer and they realize the full potential network value, S. Hence, no other situation can Pareto-dominate this corner equilibrium. The corner equilibrium in which everyone chooses good 1 will be cost benefit optimal if $N_A D_A \geq N_B D_B$, and the other corner equilibrium will be cost-benefit optimal if $N_B D_B \geq N_A D_A$.

We have then the familiar equilibrium selection problem associated with network externalities. There are two equilibria, both of which are Pareto-optimal, but to achieve either requires that the choices of all individuals be somehow coordinated. In the simultaneous move game there is no obvious means of coordination, and hence no assurance that either equilibrium will be achieved.

In this paper we assess the possibilities for coordination in a framework in which individuals choose good 1 or good 2 in sequence, knowing only their own type, the choices made by those who precede them in the choice making sequence, the taste parameters of the model, and the total number of individuals in the choice making sequence.

3 The Sophisticated Individual's Choice Problem

Consider the decision problem of individual k in the choice making sequence. By hypothesis, she knows the number of people who previously chose good 1, M_1, and the number who previously chose good 2, M_2 (where $M_1 + M_2 = k - 1$); she knows her own type, A or B; she knows the total number of people in the choice making sequence, N; and she knows the five taste parameters of the model, S, z_{A1}, z_{A2}, z_{B1}, and z_{B2}. Although we distinguish five taste parameters, there are in fact just three relevant taste parameters, S, D_A, and D_B.

Since her realized network value is dependent upon the number of other individuals who make the same choice that she makes, she must form subjective probabilities concerning the total number of people who will eventually choose good 1. (Since everyone chooses one good or the other, we can focus on expectations with respect to the number of people who eventually choose good 1.) Of course, the choices of those who follow her in the sequence may depend upon her own choice, so she needs two sets of subjective probabilities, which correspond to the two goods she might choose. Define $G_k(Q_1; M_1, i)$ as person k's subjective probability that a total of Q_1 individuals other than k eventually choose good 1, given that M_1 of the first $k-1$ individuals have chosen good 1, and given that individual k chooses good i. With $N - k$ individuals following individual k in the choice making sequence, $M_1 \leq Q_1 \leq M_1 + N - k$.

Let z_{k1} and z_{k2} denote the private consumption values of individual k, where (z_{k1}, z_{k2}) is either (z_{A1}, z_{A2}) or (z_{B1}, z_{B2}). Individual k will choose good 1 if the following inequality is satisfied:

$$z_{k1} + \frac{S}{N-1} \cdot \sum_{Q_1=M_1}^{M_1+N-k} Q_1 \cdot G_k(Q_1; M_1, 1) >$$

$$z_{k2} + \frac{S}{N-1} \cdot \sum_{Q_1=M_1}^{M_1+N-k} Q_2 \cdot G_k(Q_1; M_1, 2), \tag{1}$$

where $Q_2 = N - 1 - Q_1$, and will choose good 2 if the inequality is reversed.

To solve this choice problem we must develop a calculus of subjective probability; that is, we must develop the function $G_k(Q_1; M_1, i)$, and then use it to solve person k's choice problem. Since these subjective probability distributions are, in effect, a theory about the choices of the individuals who follow individual k in the choice making sequence, the natural way to proceed is use a dynamic programming approach, beginning with the choice problem of the last individual in the choice sequence, and working our way back to individual k.

Before doing so, it is useful to define what we mean by a *coordination cascade.* If M_1, is large relative to $k - 1 - M_1$, and if S is large enough, it will be the case that individual k will choose good 1 regardless of her type, and that all individuals who follow her in the choice making sequence will also choose good 1, regardless of their type, and we say that individual k is trapped in a *good 1 coordination cascade.* Alternatively, if M_1 is small relative to $k-1-M_1$, and if S is large enough, individual k is trapped in a *good 2 coordination cascade.* Individuals not trapped in a coordination cascade will simply follow their private preferences, type A individuals will choose good 1 and type B will choose good 2. If a coordination cascade arises early in the choice sequence, then despite the lack of a coordinator, most individuals will choose one good or the other, and we will end up at an equilibrium of the sequential choice game that is close to one of the equilibria of the simultaneous choice game.

Once we have an understanding of coordination cascades, it is clear that to solve person k's choice problem we must find the largest integer value of M_1 such that person k chooses good 2 regardless of her type, which we denote by Δ_k, and the smallest integer value of M_1 such that person k chooses good 1 regardless of her type, which we denote by Λ_k. We call the pair (Δ_k, Λ_k) person k's *policy*, and we call the entire collection of N policies a *policy set*, (Δ, Λ).

We start with the choice problem of the last person in the choice sequence, person N. Notice that, since she is the last person, $Q_1 = M_1$, and hence,

$$G_N(M_1; M_1, 1) = G_N(M_1; M_1, 2) = 1. \tag{2}$$

As defined above, Δ_N is the largest (integer) value of M_1 such that person N chooses good 2, regardless of her type; that is, even if her type is A. Then, using (1) and (2), we see that Δ_N is the largest (integer) value of M_1 such that

$$z_{A2} + \frac{S}{N-1} \cdot (N - 1 - M_1) \geq z_{A1} + \frac{S}{N-1} \cdot M_1. \tag{3}$$

But this implies that

$$\Delta_N = INT\left[\left(\frac{N-1}{2}\right) \cdot \left(1 - \frac{D_A}{S}\right)\right], \tag{4}$$

where $INT(x)$ is the function that picks the largest integer in x. Similarly,

$$\Lambda_N = INT\left[\left(\frac{N-1}{2}\right) \cdot \left(1 + \frac{D_B}{S}\right)\right]. \tag{5}$$

Notice that $\Delta_N < INT[(N-1)/2]$ since $S > D_A$ and that Δ_N increases as S increases. Similarly, $\Lambda_N > INT[(N-1)/2]$ and Λ_N decreases as S increases.

Person $N-1$ will use M_1, her information about her own type, and knowledge of (Δ_N, Λ_N) to form her own subjective probability distributions $G_{N-1}(Q_1; M_1, i)$. Consider first $G_{N-1}(Q_1; M_1, 1)$, the subjective probability distribution that is associated with the event that person $N-1$ chooses good 1. As explained below

1. if $M_1+1 \leq \Delta_N$, then $G_{N-1}(M_1; M_1, 1) = 1$;

2. if $M_1+1 \geq \Lambda_N$, then $G_{N-1}(M_1+1; M_1, 1) = 1$;

3. if $\Delta_N < M_1+1 < \Lambda_N$,

   then $G_{N-1}(M_1+1; M_1, 1) = \frac{M_1+1}{N-1}$,

   and $G_{N-1}(M_1; M_1, 1) = \frac{N-M_1-2}{N-1}$.

If $M_1+1 \leq \Delta_N$ and person $N-1$ chooses good 1, then person N will be in a good 2 coordination cascade, and $Q_1 = M_1$ with probability 1. Similarly, if $M_1+1 \geq \Lambda_N$ and person $N-1$ chooses good 1, person N will be in a good 1 coordination cascade and $Q_1 = M_1+1$ with probability 1. If $\Delta_N < M_1+1 < \Lambda_N$, then person N will be in neither sort of cascade if person $N-1$ chooses good 1, and hence person N's type will determine her choice. If person $N-1$ is type A, the information available to her (that she, and M_1 of the first $N-2$ persons in the choice sequence are type A) yields a subjective probability that person N is type A equal to $(M_1+1)/(N-1)$ and a subjective probability that person N is type B equal to $(N-M_1-2)/(N-1)$, as reported in 3 above. Of course, if person $N-1$ was type B her subjective probability that person N is type A would be $M_1/(N-1)$, but this subjective probability is irrelevant since person $N-1$ will choose good 1 only if she is type A, since she is in neither sort of cascade.

We can find $G_{N-1}(Q_1; M_1, 2)$ in just the same way.

1. if $M_1 \leq \Delta_N$, then $G_{N-1}(M_1; M_1, 2) = 1$;

2. if $M_1 \geq \Lambda_N$, then $G_{N-1}(M_1+1; M_1, 2) = 1$;

3. if $\Delta_N < M_1 < \Lambda_N$,

then $G_{N-1}(M_1+1; M_1, 2) = \frac{M_1}{N-1}$,

and $G_{N-1}(M_1; M_1, 2) = \frac{N-1-M_1}{N-1}$.

These subjective probability distributions in turn allow us to find the policy of person $N-1$, $(\Delta_{N-1}, \Lambda_{N-1})$. By definition, Δ_{N-1} is the largest integer M_1 such that

$$z_{A2} + \frac{S}{N-1} \cdot \sum_{Q_1=M_1}^{M_1+1} Q_1 \cdot G_{N-1}(Q_1; M_1, 2) >$$

$$z_{A1} + \frac{S}{N-1} \cdot \sum_{Q_1=M_1}^{M_1+1} Q_2 \cdot G_{N-1}(Q_1; M_1, 1). \tag{6}$$

Similarly, Λ_{N-1} is the largest integer M_1 such that

$$z_{B1} + \frac{S}{N-1} \cdot \sum_{Q_1=M_1}^{M_1+1} Q_1 \cdot G_{N-1}(Q_1; M_1, 1) >$$

$$z_{B2} + \frac{S}{N-1} \cdot \sum_{Q_1=M_1}^{M_1+1} Q_2 \cdot G_{N-1}(Q_1; M_1, 2). \tag{7}$$

We have then all that is needed to compute person $(N-1)$'s policy, $(\Delta_{N-1}, \Lambda_{N-1})$.

Person $N-2$ can use M_1 (the number of persons who previously chose good 1), her information about her own type, and knowledge of (Δ_N, Λ_N) and $(\Delta_{N-1}, \Lambda_{N-1})$ to form her own subjective probability distributions $G_{N-2}(Q_1; M_1, i)$. This in turn will allow person $N-2$ to solve her own choice problem. This solution, of course, is described by her policy, $(\Delta_{N-2}, \Lambda_{N-2})$. Then, building on these results, we can find the policy of person $N-3$, and so on.

It is tedious to actually compute these policies, and it becomes increasingly tedious to do so as we move further up the choice making sequence. For this reason, we have relied on numerical techniques to generate the policy set (Δ, Λ) for particular configurations of the model's parameters, S, N, D_A, and D_B.

In Figure 1 we picture the policy set for the case in which $N = 30$, and $S = 2.5 \cdot D_A = 2.5 \cdot D_B$, so the potential network value is two and a half times as large as the private preference differentials. The choice making sequence proceeds from top to bottom in the figure. For each person k, the horizontal distance from one side of the triangle to the other is equal to $k-1$, and it therefore serves as an axis for M_1 for that person. The jagged line on the left connects the Δ_ks and the one on the right the Λ_ks. If person k finds herself in a situation where M_1 is on or to the left of the first jagged line, she and all who follow her are in a good 2 cascade, and if she finds herself in a situation where M_1 is on or to the right of the second jagged line, she and all who follow her are in a good 1 cascade. And if she finds herself in a situation where M_1 lies between the jagged lines, she chooses the good that she privately prefers.

The policy set itself is reported at the bottom of Figure 1. Persons 1, 2 and 3 in the choice sequence will all follow their private preferences. Person 4 will be trapped in a good 2 cascade if none or just one of the first three people choose good 1, and she will be trapped in good 1 cascade if two or three of the first three people choose good 1. This, of course, means that person 4 will definitely be trapped in one cascade or the other. Since everyone who follows her will also be in cascade, just four terminal states are possible. We can describe a terminal state by the number of persons in total who choose good 1, T_1, where T_1 is an integer in the set $\{0, 1, ..., N\}$. If the first three people choose good 1, that is, if the first three people are all type A, then the fourth is trapped in a good 1 cascade and $T_1 = 30$. Analogously, if the first three all choose good 2, $T_1 = 0$. If two of the first three people choose good 1 (respectively good 2), then the fourth is again trapped in a good 1 cascade (respectively good 2), and $T_1 = 29$ (respectively, $T_1 = 1$). In this case, coordination is important since $S = 2.5 \cdot D_A = 2.5 \cdot D_B$, and because it is so important, the fourth person in the choice sequence is inevitably trapped in a cascade. The type of cascade that emerges and the terminal state that is achieved depend on the types of the first three persons in the choice sequence.

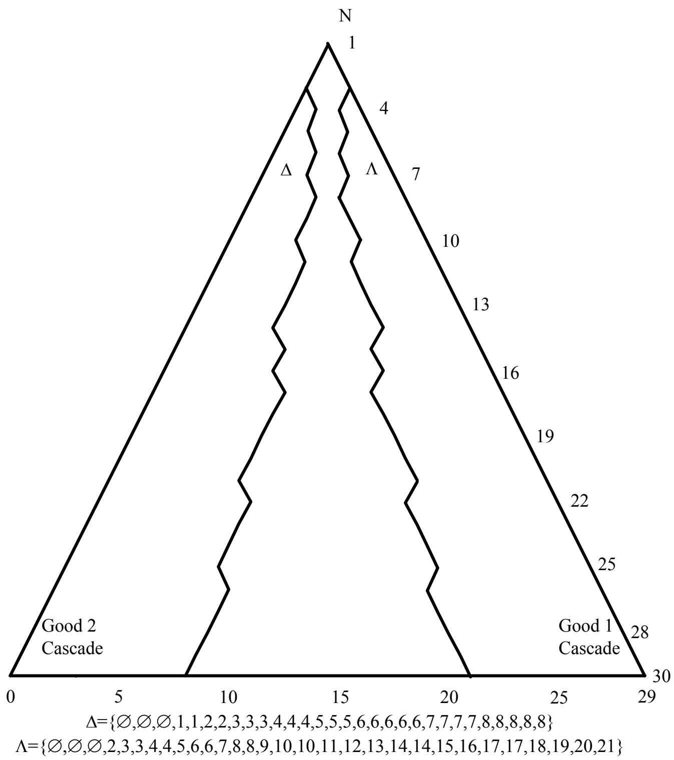

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Figure 1: Policy Set for $S = 2.5 \cdot D_A = 2.5 \cdot D_B$

In Figure 2 we picture the policy set for the case in which $N = 30$, and $S = 1.5 \cdot D_A = 1.5 \cdot D_B$. Notice that the first opportunity for a cascade does not arise until the sixth person makes her choice, and she will be trapped in a cascade if and only if at least four of the first five people make the same choice. If a cascade does not arise when the sixth person makes her choice, the next opportunity will not arise until the tenth person makes her choice, and she will be in a cascade if and only if at least seven of the first nine people are of the same type. With these parameter values coordination is less important, and is very difficult to achieve.

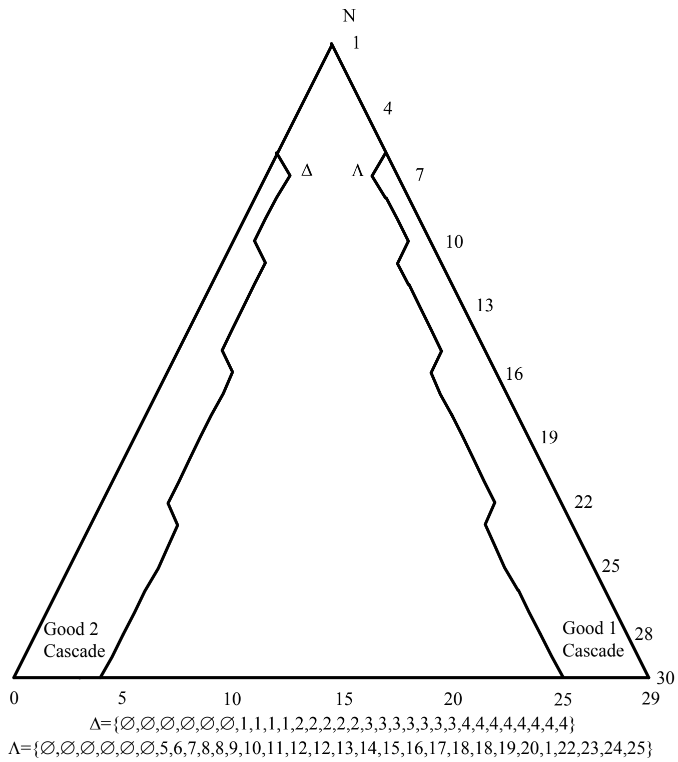

Figure 2: Policy Set for $S = 1.5 \cdot D_A = 1.5 \cdot D_B$

4 The Naïve Individual's Choice Problem

We have shown that if the network externality is sufficiently strong, individuals who are sophisticated enough to perform the calculations envisioned in our model of sequential choice manage to achieve substantial coordination. As a point of comparison, it is useful to compare these results with the corresponding results for naïve individuals. Naïve individuals, by definition, do not take into account the effects that their adoption decisions maight have on the adoption decisions of people who follow them in the choice making sequence. Instead, they behave as if they were the last person in the sequence.

Since naive individuals do not anticipate cascades, they do not form subjective probabilities concerning the total number of people who will eventually choose good

1. However, coordination cascades can still arise, and therefore we must still calculate the naive individuals's policy. In solving person k's choice problem we find the largest integer value of M_1 such that person k chooses good 2 regardless of her type, which we now denote δ_k, and the smallest integer value of M_1 such that person k chooses good 1 regardless of her type, which we now denote λ_k. We call the pair (δ_k,λ_k) naïve person k's policy and we call the entire collection of N policies, (δ, λ), a naïve policy set.

Since naïve individuals behave as if they aer the last person in the sequence, naïve individual K will choose good 2 even if she is type A if the following inequality holds:

$$z_{A2} + \frac{S}{K-1} \cdot (K - 1 - M_1) \geq z_{A1} + \frac{S}{K-1} \cdot M_1. \tag{8}$$

This implies that

$$\delta_K = INT\left[\left(\frac{K-1}{2}\right) \cdot \left(1 - \frac{D_A}{S}\right)\right] \tag{9}$$

and similarly,

$$\lambda_K = INT\left[\left(\frac{K-1}{2}\right) \cdot \left(1 + \frac{D_B}{S}\right)\right]. \tag{10}$$

In Figure 3 we picture the naïve policy set for the case in which $N = 30$, and $S = 2.5 \cdot D_A = 2.5 \cdot D_B$. It is now the case that persons 1, 2, 3 and 4 in the choice sequence will all follow their private preferences. Person 5 will be trapped in a good 2 cascade if none or just one of the first four people choose good 1, and she will be trapped in a good 1 cascade if three or four of the first four people choose good 1. However, if only two of the first four people choose good 1, then person 5 follows their private preferences. In this case, it is no longer guaranteed that a cascade will form, unlike the situation where individuals take into account the future effect of their adoption decisions. In fact, for naïve individuals the probability of a coordination cascade forming is always smaller than for sophisticated consumers.

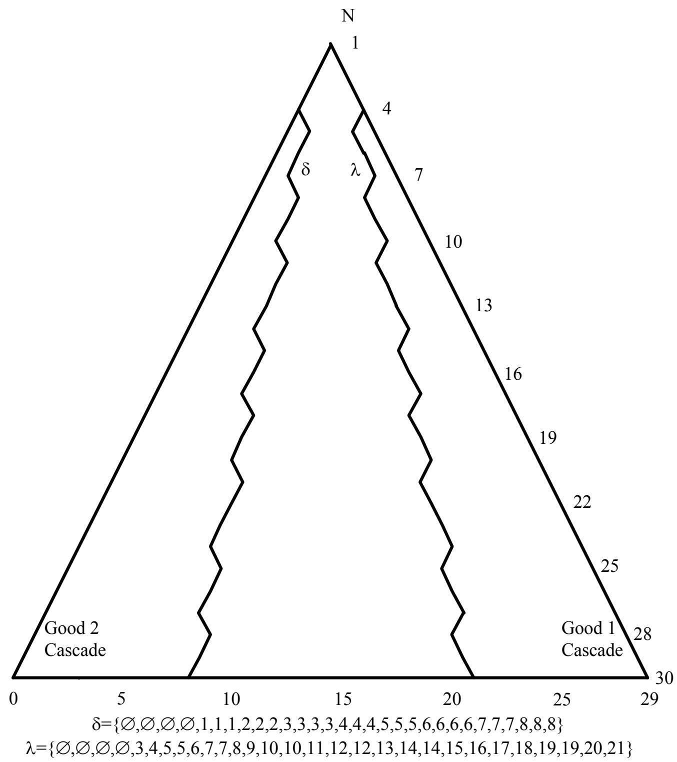

Figure 3: Naive Policy Set for $S = 2.5 \cdot D_A = 2.5 \cdot D_B$

5 Aggregate Results

In this section we examine aggregate choices for groups of individuals when the sequence in which the individuals make their choices is random. We report the probability distributions over terminal states and a measure of relative efficiency.

We use the following approach:

1. create a group of individuals by choosing values for the following parameters – S, N_A, N_B, D_A, and D_B;
2. find the policy set for the given parameter values – the policy set depends on the total number of people, $N = N_A + N_B$, but not on the numbers of people of types A and B, and the preference parameters;

3. generate a frequency distribution over terminal states by choosing a large number (10,000 is the number used in the simulations reported below) of random choice sequences for the group, and for each random sequence use the policy set to determine the associated terminal state – the frequency distribution, in turn, generates an approximate probability distribution over terminal states.

We calculate relative efficiency as follows. Let U^* denote total utility in the cost-benefit optimal equilibrium of the simultaneous choice game, U^p denote total utility when individuals follow their private preferences, U^T denote total utility in terminal state T, and p^T the approximate probability of terminal state T. Relative efficiency (RE), is then defined as follows:

$$RE = \frac{\left(\sum_T p^T U^T\right) - U^P}{U^* - U^P}. \tag{11}$$

Notice that $0 \leq RE \leq 1$. The denominator in this expression is the maximum possible gain in utility, relative to the situation where people simply follow their private preferences, and the numerator os the expected gain in utility relative to the same point of comparison. Thus, RE measures the expected proportion of the maximal gain in utility from achieving perfect coordination that is achieved in our sequential choice model.

Results for the case in which $S = 2.5$, $N_A = 15$, $N_B = 15$, $D_A = 1$, and $D_B = 1$ are reported in Figure 4. Recall that the policy set for this case is pictured in Figure 1. For purposes of comparison, in the bottom half of the figure we report the frequency distribution over terminal states, $g(M)$, that would arise if individuals simply chose the good that they privately prefer. Since there are 30 people in the group and half of them are type A, the terminal state that is generated when people follow their private preferences is $T_1 = 15$. In the top half of the figure we report the frequency distribution that is generated when individuals use the policy set to make their choices. As we saw earlier, with this policy set the fourth person inevitably finds herself in a coordination cascade, and there are just four possible terminal states: $T_1 = 0$, $T_1 = 1$, $T_1 = 29$, and $T_1 = 30$. In our simulations, the relative frequency of these terminal states is 0.115, 0.386, 0.388, and 0.110.[4] The relative efficiency for this case is 0.87, so the failure to achieve perfect coordination in this case results in an expected aggregate increase in welfare, relative to the welfare achieved when individuals follow their private preference, that is 87% of the maximal increase.

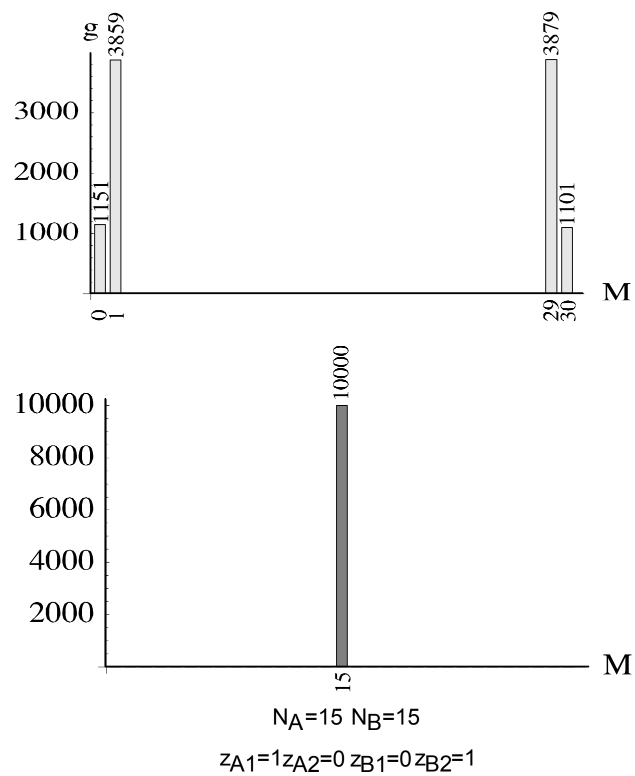

Figure 4: Results for $S = 2.5 \cdot D_A = 2.5 \cdot D_B$

To get a feeling for the comparative statics of this model with respect to S, in

[4]For this case we can easily calculate the actual probabilities of these terminal states, which serves as a very rough check on the accuracy of our approximation. Terminal states $T_1 = 0$ and $T_1 = 30$ require that the first three individuals in the choice sequence be of the same type, an event that will occur with probability $(15/30) \cdot (14/29) \cdot (13/28)$ which is approximately 0.112. Terminal states $T_1 = 1$ and $T_1 = 29$ require that exactly two of the first three persons in the choice sequence be of the same type, which will occur with probability $3 \cdot (15/30) \cdot (14/29) \cdot (15/28)$, which is approximately 0.388.

Figures 5 and 6 we hold the private taste parameters and numbers of individuals of types A and B constant, and reduce S to 2.0 in Figure 5, and to 1.5 in Figure 6. As can be seen from the figures, as S falls, the probability of a cascade falls, to 0.84 in Figure 5 and to 0.18 in Figure 6, and so to does relative efficiency. In Figure 5 relative efficiency is 0.65 and it is 0.14 in Figure 6.

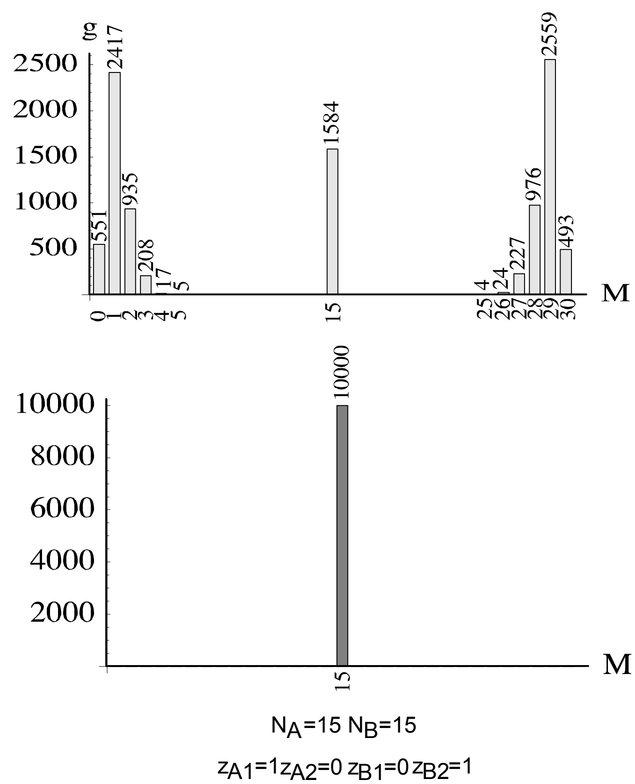

Figure 5: Results for $S = 2.0 \cdot D_A = 2.0 \cdot D_B$

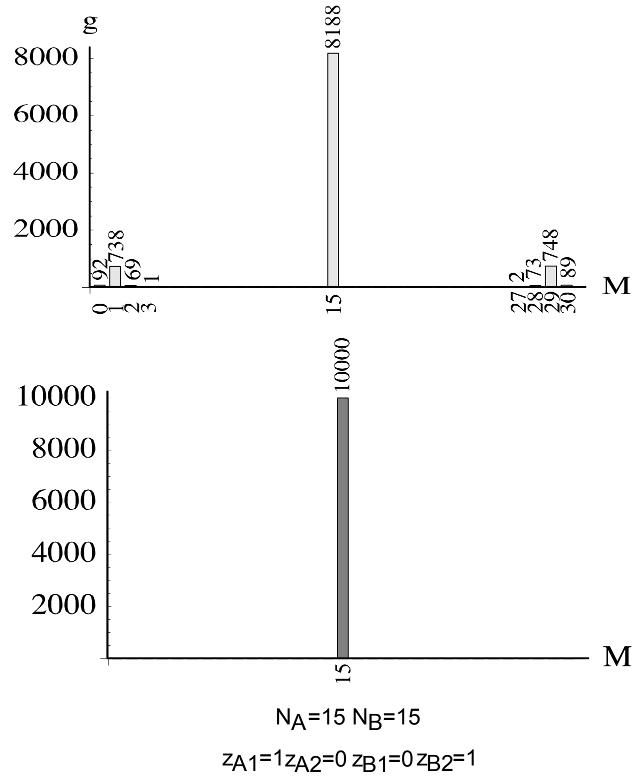

Figure 6: Results for $S = 1.5 \cdot D_A = 1.5 \cdot D_B$

In tables 1 and 2 we present further results concerning the degree to which coordination is achieved, and expected relative efficiency as S varies, holding private preference parameters and the numbers of individuals of each type constant at the values specified above. Table 1 shows that coordination cascades begin to appear when S is approximately 1.3. The probability of a cascade then increases quickly as S increases. For S greater than approximately 2.1, the probability of a cascade is 1.[5] The message is clear and intuitive: as coordination becomes more important, that is, as S increases, it is more readily achieved.

The reason for the quick transition between no cascade forming and a cascade always forming, is because cascades arise early in the sequence of choices. Since individuals take into account what the individuals who follow them might choose,

[5]Note that for naïve individuals, for the results presented, the probability of a cascade forming is less than 1.

individuals at the beginning of the sequence will be more willing to start a cascade then they would if they did not take future actions into account. Recall the discussions of Figures 1 and 2. Figure 2 shows the policy set for $S = 1.5$. In that case, the opportunity of a cascade does not arise until person 6. In contrast, in Figure 1, cascades are inevitable (since the forth person is always in one cascade or the other). As the network value S increases, the distance between the policy pairs decreases thereby increasing the chances for a cascade forming early in the sequence of choices.

Table 1: Probability of Cascade wrt S

	S							
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
Sophisticated	0	0	0	0.03	0.09	0.18	0.35	0.40
Naïve	0	0	0	0.01	0.04	0.17	0.17	0.34
	S							
	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
Sophisticated	0.70	0.72	0.84	1.0	1.0	1.0	1.0	1.0
Naïve	0.36	0.36	0.65	0.64	0.65	0.65	0.71	0.72

Table 2 tells us what happens to relative efficiency as S increases. At $S = 1$, cascades never occur as individuals always follow their private preferences, and hence relative efficiency is zero. As S increases, relative efficiency increases in step with the increase in the probability of a cascade forming.

Table 2: Relative Efficiency wrt S								
	S							
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
Sophisticated	0	0	0	0.02	0.06	0.14	0.27	0.31
Naïve	0	0	0	0.01	0.03	0.13	0.13	0.27
	S							
	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
Sophisticated	0.56	0.57	0.65	0.85	0.86	0.86	0.87	0.87
Naïve	0.29	0.29	0.53	0.53	0.54	0.54	0.58	0.59

The case illustrated in Figure 5 serves as a baseline for the comparative static exercises reported below. In Figure 7 we increase the number of individuals of each type to 50, thus increasing the size of the group from 30 to 100, while holding the other parameters constant. The probability of a cascade increases from 0.84 to 0.98, and relative efficiency increases from 0.65 to 0.89.

N_A=50 N_B=50
z_{A1}=1 z_{A2}=0 z_{B1}=0 z_{B2}=1

Figure 7: Results for $N = 100$ and $S = 2.0$

Tables 3 and 4 report comparative statics concerning the degree to which coordination is achieved, and expected relative efficiency as the total number of individuals varies, holding the network value, private preference parameters, and the numbers of each type of individuals constant. Table 3 indicates that the probability of a cascade increases as the size of the population increases, and table 4 indicates that relative efficiency also increases as the size of the population increases. The message is clear: larger groups are better able to achieve coordination in our sequential choice framework.

Table 3: Probability of Cascade wrt N										
	N									
	10	20	30	40	50	60	70	80	90	100
Sophisticated	0.53	0.69	0.84	0.87	0.93	0.94	0.95	0.97	0.98	0.98
Naïve	0.53	0.62	0.63	0.64	0.65	0.65	0.66	0.67	0.67	0.67

Table 4: Relative Efficiency wrt N										
	N									
	10	20	30	40	50	60	70	80	90	100
Sophisticated	0.28	0.49	0.65	0.71	0.78	0.81	0.83	0.86	0.88	0.89
Naïve	0.28	0.46	0.52	0.56	0.58	0.59	0.61	0.62	0.63	0.63

In Figure 8, we change the composition of the population, reducing the number of type A persons from 15 to 9 and increasing the number of type B persons from 15 to 21. The cost benefit criterion now favors coordination on good 2, and relative to Figure 4 we see a couple of significant things. The probability of a cascade is now 0.96, and the probability of a good 2 cascade is now 0.88. In addition, expected relative efficiency increases form 0.65 to 0.80.

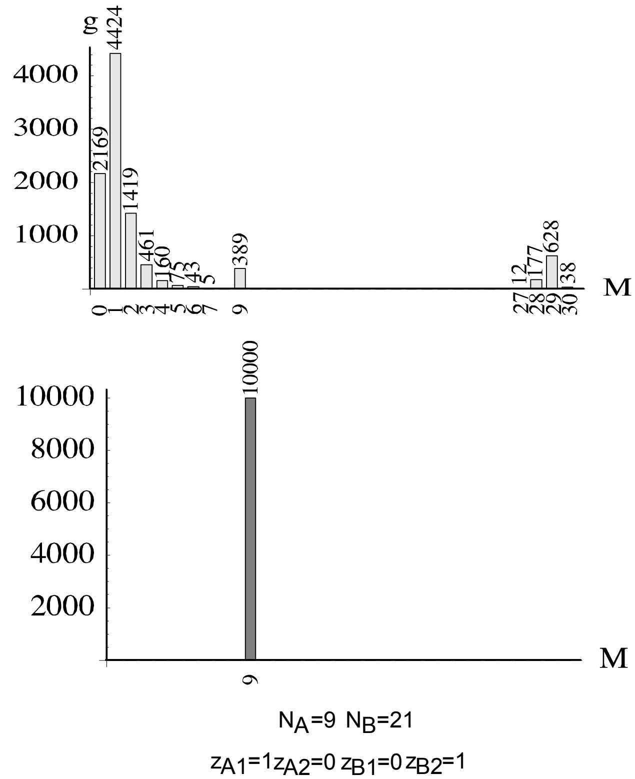

Figure 8: Results for $N_A = 9, N_B = 21, S = 2.0$

Tables 5 and 6 illustrate the comparative statics in relation to the degree to which coordination is achieved, and the expected relative efficiency as the composition of the population varies, holding the network value, total number of individuals and private preference parameters constant. Notice that coordination is easier to achieve when the population consists of more like-minded individuals. For example, when the number of type B individuals is 27 out of 30, the probability of a cascade is essentially 1. As well, since coordination is easier to achieve in this case, expected relative efficiency is also high, 0.88.[6] Again, the message is clear: in our sequential

[6] The relative efficiency results for naïve individuals is due to the relationship between which type of cascade forms and the number of each type of individual. When $N_A/N = 0.1$, the probability of a good 2 cascade is less, and the probability of no cascade is higher, relative to the situation with sophisticated individuals. In the naïve scenario, more individuals are choosing their privately preferred good, and thus, expected total utility is higher. However, the gain is not large because of the small number of type A individuals. When $N_A/N = 0.3$, the gain is the largest. At $N_A/N = 0.5$, the relationship between the type of cascade and number of each type of individual balances out and relative efficiency decreases (i.e. the number of individuals not consuming their privately preferred

choice framework, asymmetric populations achieve coordination more readily.

Table 5: Proability of Cascade wrt N_A/N

	N_A/N				
	0.1	0.2	0.3	0.4	0.5
Sophisticated	0.99	0.99	0.96	0.89	0.84
Naïve	0.99	0.98	0.86	0.71	0.64

Table 6: Relative Efficiency wrt N_A/N

	N_A/N				
	0.1	0.2	0.3	0.4	0.5
Sophisticated	0.88	0.81	0.80	0.75	0.65
Naïve	0.90	0.83	0.95	0.85	0.52

In Figure 9, we change the private preference differential for type B individuals, D_B, from 1 to 2.0. Since $D_A = 1 < D_B = 2.0$, type A individuals are less inclined to follow their own preferences than are type B. This in turn implies that type A individuals are more inclined to join a good 2 cascade than are type B individuals to join a good 1 cascade. This is evident in Figure 9 which shows that relative to the baseline case, the probability of a good 1 cascade has decreased from 0.42 to 0.

good increases).

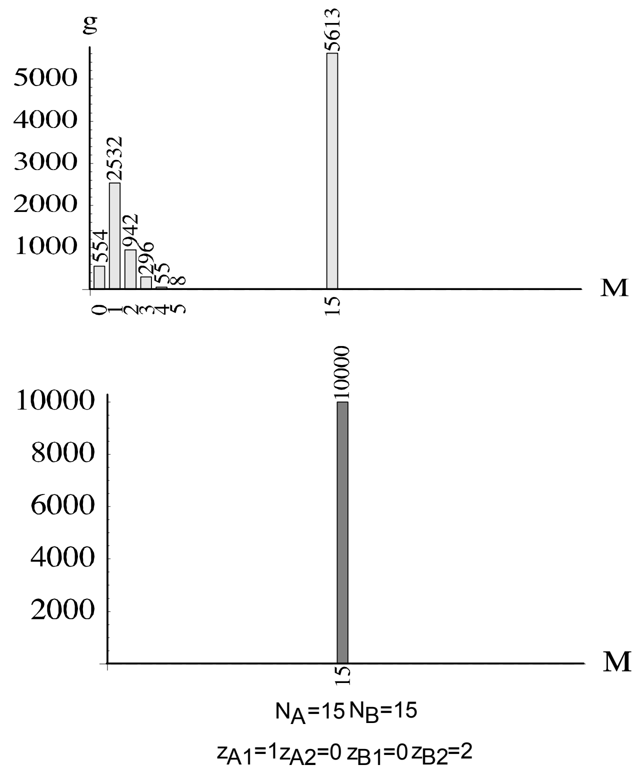

Figure 9: Results for $D_B = 2.0$ and $S = 2.0$

Table 7 illustrates the degree to which coordination is achieved as the private preference differential for type B individuals varies, holding the network value, total number of individuals and composition of the population constant. As D_B increases, type B individuals are less likely to switch from following their private preferences, and once $D_B > S$, ($S = 2$ in table 7), the private consumption values drive the choice of an individual of type B. For $D_B > S$, good 1 cascades never form and individuals will either follow a good 2 cascade or follow their private preferences. Since good 1 cascades do not form, the probability of a cascade forming also declines. Table 7 shows that the value of D_B such that type B individuals never switch is 1.9 and the probability of a cascade decreases to 0.44.

While the probability of a cascade forming does not vary substantially, the type of cascade does. As D_B increases, type B individuals are less likely to switch from following their private preferences. This implies that the probability of a good 1 cascade should decrease as D_B increases. Table 7 shows that as D_B increases, the probability of a good 1 cascade forming decreases from 0.45 at $D_B = 1$ to 0.31 at $D_B = 1.8$, and then falls to 0 at $D_B = 1.9$.[7]

TABLE 7: PROBABILITY OF CASCADE BY TYPE OF CASCADE

	D_B										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Sophisticated	0.84	0.84	0.84	0.84	0.84	0.84	0.82	0.81	0.72	0.44	0.44
Good 1	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.42	0.31	0	0
Good 2	0.42	0.42	0.42	0.42	0.42	0.42	0.39	0.39	0.41	0.44	0.44
Naïve	0.63	0.50	0.48	0.41	0.36	0.34	0.33	0.33	0.32	0.33	0.33
Good 1	0.31	0.18	0.17	0.09	0.04	0.02	0	0	0	0	0
Good 2	0.32	0.32	0.31	0.32	0.32	0.32	0.33	0.33	0.32	0.33	0.33

Finally, table 8 tells us what happens to relative efficiency as D_B increases. Given the amount of coordination that occurs, the measure of relative efficiency is small, ranging from 0.34 when $D_B = 1.9$ to 0.65 at $D_B = 1$. The reason for the low values of relative efficiency is due to the amount of coordination on good 1 that still occurs. Good 1 has a lower average private preference value than good 2, and therefore maximum utility is achieved when everyone coordinates on good 2.[8] Since good 1 cascades can still form, the realized gain in utility from coordination is not maximized. Therefore, while there still exists a substantial amount of coordination,

[7]For naïve individuals the probability of a good 1 cascade forming declines quickly. The reason is that naïve type B individuals do not forecast the future benefits of joining a good 1 cascade, and the benefits of choosing good 1 when it is their turn to choose due not outwiegh their strong private preference for good 2.

[8]When private preference values are symmetric, maximum utility can be achieved through perfect coordination on either good.

there is no guarantee that individuals will coordinate on the good that yields the greater total utility.

Table 8: Relative Efficiency wrt D_B

	D_B										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Sophisticated	0.65	0.61	0.57	0.53	0.50	0.45	0.42	0.38	0.35	0.34	0.34
Naïve	0.52	0.39	0.37	0.31	0.29	0.27	0.27	0.27	0.26	0.27	0.27

6 Conclusion

In the network externality literature, perfect coordination of consumer choices on one option is optimal, yet it is clear that in the absense of some sort of central direction, it is highly unlikely that perfect coordination will, in fact, be achieved. The question of interest would then seem to be: Given that consumer decisions are not centrally directed, what are the costs of the failure to achieve perfect coordination? We address this question in a sequential choice framework in which consumers observe the choices of those who precede them and anticipate the choices of those who follow them.

The most interesting comparative static results relate to the magnitude of the network externality and the size of the group. Both the probability of a coordination cascade, in which a large majority of consumers choose the same good, and the expected relative efficiency of the sequential choice equilibrium, increase as the magnitude of the network externality increases and as group size increases. In both cases, the probability of a cascade rapidly approaches 1, and relative efficiency increases to the 85% range. Thus, when it really counts, unfettered and undirected choice making effectively solve the coordination and equilibrium selection problems that are raised by the network externality in our model.

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