Decision Theory in Economics: an overview of where we are and where we're going

Jacob S. Sagi

Choice Theory

- Why is it important?
  - Sets the 'laws of behavior' of the principle actors in social science
    - Games
      - Strategic actors
      - What are their objectives, how do they use information
    - General equilibrium
      - Prices + information feed into choices
      - **Choices determine aggregate demand for physical and financial goods and**
      - Aggregate demand determines production (supply) + planning + prices

Aim of this talk

- Provide a basic review of the neo-classical paradigm for modeling decision making under uncertainty, and through time;
- Summarize the multiple empirical challenges facing the neo-classical paradigm;
- Highlight some of the alternatives to the classical theory that are now explored in the literature.

The neoclassical paradigm

- Probabilistic sophistication
  - Decision makers (DMs) associate unique subjective probabilities with distinct non-deterministic future events, and update using Bayes' rule
  - When offered a profile of 'payoffs in various disjoint events', the only choice-relevant attribute of an event is its probability.
- Expected Utility

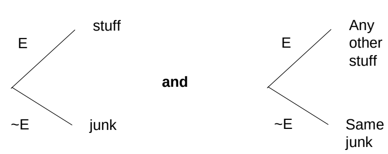
$f = (E_1, x_1; \ldots; E_n, x_n)$, $g = (E'_1, y_1; \ldots; E'_m, y_m)$

$$f \succ g \Leftrightarrow \sum_{i=1}^{n} p_i u(x_i) > \sum_{i=1}^{m} q_i u(y_i)$$

- Time consistency (backward induction)
  - Current choices correctly take into account future choices

The neoclassical paradigm

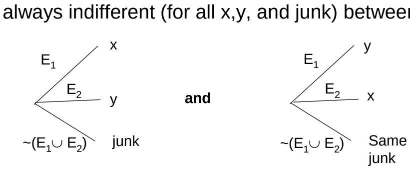
- Derives from 'behavioral axioms'
  - Primitive assumptions over choice behavior
  - Let's see how this works….
    - What can we say about the subjective likelihood of an event, E, if the DM is *always* indifferent between



- Say that E is *null*

Probabilistic sophistication

- What can we say about two events, E_1 and E_2, if the DM is always indifferent (for all x,y, and junk) between



- Say that E_1 and E_2 are exchangeable: $E_1 \approx E_2$

Probabilistic sophistication

- Behavioral axioms:
  - Preference is transitive and non-trivial
  - (Axiom A) If E_1 is not null, then any sequence of pairwise disjoint events $E_1 \approx E_2 \approx E_3$ … must be finite.
  - (Axiom N) If E_1, E_2 and A are pairwise disjoint, $E_1 \approx E_2$, and A is not null, then no subevent of E_2 is exchangeable with $E_1 \cup A$.
  - (Axiom C) If E_1 and E_2 are disjoint, then one contains a subevent that is exchangeable with the other.
- Implication (Chew & Sagi, 2006): A, N, & C iff there is a unique probability measure, m, such that
  - $m(E_1) = m(E_2) \Leftrightarrow E_1 \approx E_2$
  - $m(E_1) > m(E_2) \Leftrightarrow E_1$ contains a subevent that is exchangeable with E_2
  - $m(E_1) = 0$ iff E_1 is null
  - DM only cares about the probability of an event when assigning events to payoffs.

Utility representation

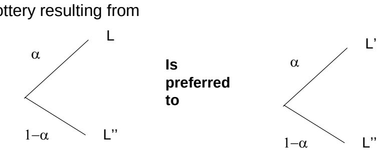
- DM treats event-payoff profiles as lotteries

  f = (E_1, x_1; …; E_n, x_n) treated as L_f = (p_1, x_1; …; p_n, x_n)
- Add Continuity Axiom (assuming appropriate topology)
  - There is a continuous utility representation,

$$f \succ g \Leftrightarrow V(p_1, x_1;...; p_n, x_n) > V(q_1, y_1;...; q_n, y_m)$$

Expected Utility

- Independence Axiom
  - If the lottery L is preferred to the lottery L', then the compound lottery resulting from



- Implication: Expected Utility

  f = (E_1, x_1; …; E_n, x_n), g = (E'_1, y_1; …; E'_m, y_m)

$$f \succ g \Leftrightarrow \sum_{i=1}^{n} p_i u(x_i) > \sum_{i=1}^{m} q_i u(y_i)$$

Time consistency

- DM's choices today reflect:
  - Full awareness of and agreement with how he will choose tomorrow
    - Rational expectations + absence of self-control problems
- Implication when choosing a multi-period consumption stream:

$$\max_{(c_1, c_2,...)\in B_1} U_1(c_1, c_2,...) = \max_{c_1 \in B'_1} W_1\left(c_1, \max_{(c_2,...)\in B_2(c_1)} U_2(c_2,...)\right)$$

- W is increasing in second argument
  - Alignment of interests

Time consistency

- Special example:
  - The exponential discounting model (time separable, time homogeneous attitudes)

$$U_t(\tilde{c}_{t+1}, \tilde{c}_{t+2},...) = E_t[u(\tilde{c}_t) + \beta U_{t+1}(\tilde{c}_{t+2}, \tilde{c}_{t+3},...)]$$

  - Iterating:

$$U_t(\tilde{c}_{t+1}, \tilde{c}_{t+2},...) = E_t[u(\tilde{c}_t)] + \beta E_t[u(\tilde{c}_{t+1})] + \beta^2 E_t[u(\tilde{c}_{t+2})] + ...$$
$$= E_t\left[\sum_i \beta^i u(\tilde{c}_{t+i})\right]$$

  - Chief attractions: simplicity

The neoclassical paradigm: recap

- Probabilistic sophistication
  - Decision makers (DMs) associate unique subjective probabilities with distinct non-deterministic future events, and update using Bayes' rule
  - When offered a profile of 'payoffs in various disjoint events', the only choice-relevant attribute of an event is its probability.
- Expected Utility

  f = (E_1, x_1; …; E_n, x_n), g = (E'_1, y_1; …; E'_m, y_m)

$$f \succ g \Leftrightarrow \sum_{i=1}^{n} p_i u(x_i) > \sum_{i=1}^{m} q_i u(y_i)$$

- Time consistency (backward induction)
  - Current choices correctly take into account future choices
    - Exponential discounting model

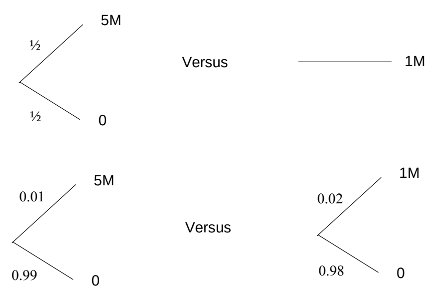
Empirical Challenges

- The Ellsberg Paradox
- The Allais Paradox
- Loss aversion and the endowment effect
- Time inconsistencies and self control

The Ellsberg Paradox

- Two urns, one ball will be drawn from each
  - Urn 1: 50 red, 50 black balls
  - Urn 2: 100 balls, red & black, unknown distribution
- If the stakes are $10k, bet on:
  - Red in urn 1 or black in urn 1?
    - Indifferent
  - Red in urn 2 or black in urn 2?
    - Indifferent
  - Red in urn 1 or red in urn 2?
    - Most people strictly prefer bets on urn 1
- Inconsistent with probabilistic sophistication
  - People are not Bayesians

The Allais Paradox



Typical choice pattern violate the Independence Axiom. People do not maximize expected utility.

Loss aversion & endowment effect

- Group A
  - Endowed with $2000
  - Asked to choose between:
    - Lose $500 for sure
    - Lose $1000 with probability ½, or nothing.
- Group B
  - Endowed with $1000
  - Asked to choose between:
    - Gain $500 for sure
    - Gain $1000 with probability ½, or nothing.
- Choice pattern is inconsistent with preferences defined *only* over consequences and likelihoods.

Time inconsistencies and self-control

- Self control
  - Procrastination
    - I'll study for the exam (or do the referee report) tomorrow….
  - Addiction
    - This is going to be my last cigarette….
- Failure of backward induction
  - Would you prefer $50 right now or $100 next week?
  - Would you prefer $50 in 52 weeks or $100 in 53 weeks?

Addressing the empirical challenges

- Maxmin multiprior and robust control models
- Cumulative prospect theory
- Preference for commitment and flexibility
- A brief mention of 'behavioral' models

Maxmin multiprior and robust control models

- Payoff profile f, pays f(ω) in state ω.
  - EU ranking

$$f \succ g \Leftrightarrow \int u(f(\omega))\, d\mu(\omega) > \int u(g(\omega))\, d\mu(\omega)$$

  - Maxmin EUT

$$f \succ g \Leftrightarrow \min_{\mu \in Q} \int u(f(\omega))\, d\mu(\omega) > \min_{\mu \in Q} \int u(g(\omega))\, d\mu(\omega)$$

    - Q is a closed & convex set of subjective probability measures
    - A payoff profile is evaluated based on 'worst case beliefs'
    - Ellsberg?
      - Known urn: all measures agree that 'red' is a 50% event
      - Unknown urn: some measures assign higher (lower) likelihood to red than to black

Maxmin multiprior and robust control models

- Maxmin EUT is a special example of preferences exhibiting 'uncertainty aversion'
  - Avoid bets in which probabilities are not objectively specified
  - Additional 'penalty' for the absence of quantifiable probabilities in a decision making situation
- Another example of this: Robust control

$$V(f) = \min_{\mu \in Q(\mu_0)} \left\{ \int u(g(\omega))\, d\mu(\omega) + \theta \int \ln \frac{d\mu}{d\mu_0}\, d\mu(\omega) \right\}$$

- μ_0 is a reference measure, Q a set of absolutely continuous measures wrt μ_0.

Cumulative prospect theory

- Allais paradox and endowment effect suggest people exhibit non-linear preferences (in probabilities) and care about gains/losses relative to a reference point
- Enter Prospect Theory
  - Gain vs. loss attitudes: easy! Just evaluate expected *relative* utility….

$$f \succ g \Leftrightarrow \int u(f(\omega) - w)\, d\mu(\omega) > \int u(g(\omega) - w)\, d\mu(\omega)$$

  - w is current wealth
  - DM only cares about changes from status quo
  - Still have Allais paradox & Ellsberg paradox….

Cumulative prospect theory

- Prospect theory can also be modified so that it accommodates Allais and Ellsberg
- The most descriptively successful theory to date
- Issues:
  - Reference dependence may lead to 'silly' behavior that is not descriptively accurate
  - E.g., prefer p over q when SQ is at r, but prefer q over p when the reference point is at p.
  - Prospect Theory violates this simple condition (Sagi, 2006)
  - Prospect theory with all the bells and whistles is also mathematically difficult
    - Not many applications in GE and games

Preference for commitment and flexibility

- We saw that DMs may not exhibit full awareness of and agreement with future choice. How can this be modeled?
- Instead of

$$\max_{(c_1, c_2, \ldots) \in B_1} U_1(c_1, c_2, \ldots) = \max_{c_1 \in B'_1} W_1\left(c_1, \max_{(c_2, \ldots) \in B_2(c_1)} U_2(c_2, \ldots)\right)$$

- How about

$$\max_{(c_1, c_2, \ldots) \in B_1} U_1(c_1, c_2, \ldots) = \max_{c_1 \in B'_1} W_1\left(c_1, \max_{(c_2, \ldots) \in B_2(c_1)} U_2^a(c_2, \ldots), \max_{(c_2, \ldots) \in B_2(c_1)} U_2^b(c_2, \ldots), \ldots\right)$$

- Interpretation:
  - U_2's correspond to possible future tastes (tastes are unpredictable)
    - If W increases in all U_2's then the DM will not want to commit to any contingent plan (dynamic program) and will value flexibility

Preference for commitment and flexibility

$$\max_{(c_1, c_2, \ldots) \in B_1} U_1(c_1, c_2, \ldots) = \max_{c_1 \in B'_1} W_1\left(c_1, \max_{(c_2, \ldots) \in B_2(c_1)} U_2^a(c_2, \ldots), \max_{(c_2, \ldots) \in B_2(c_1)} U_2^b(c_2, \ldots), \ldots\right)$$

- Interpretation:
  - U_2's correspond to possible future tastes (tastes are unpredictable)
    - If W decreases in some U_2's then the DM may wish to commit to some contingent plan (dynamic program).
    - DM may not 'like' some future realizations of tastes
      - Addiction
      - Self-control

A brief mention of 'behavioral' models

- Also motivated by experimental evidence against neoclassical paradigm
- Model driven, rather than behaviorally (axiomatically) driven
  - Psychological mechanism is hypothesized and then modeled
    - Resulting choice behavior can then be tested
  - Contrasts with axiomatic models in which only observable/testable choice behavior is hypothesized and then a model is deduced
- E.g., hypothesize that a DM plays a strategic game against future ‘selves’. Observed choice is the result of an equilibrium in this game.
- A very rich literature
  - See references

25

Conclusions

- We’ve described the neo-classical paradigm for modeling decision making under uncertainty, and through time;
- We’ve summarize the multiple empirical challenges facing the neo-classical paradigm;
- Highlight some of the alternatives to the classical theory that are now explored in the literature.
  - This is still a very active field of research

26